# Principles of Communications

## ECS 332

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## 5. Angle Modulation

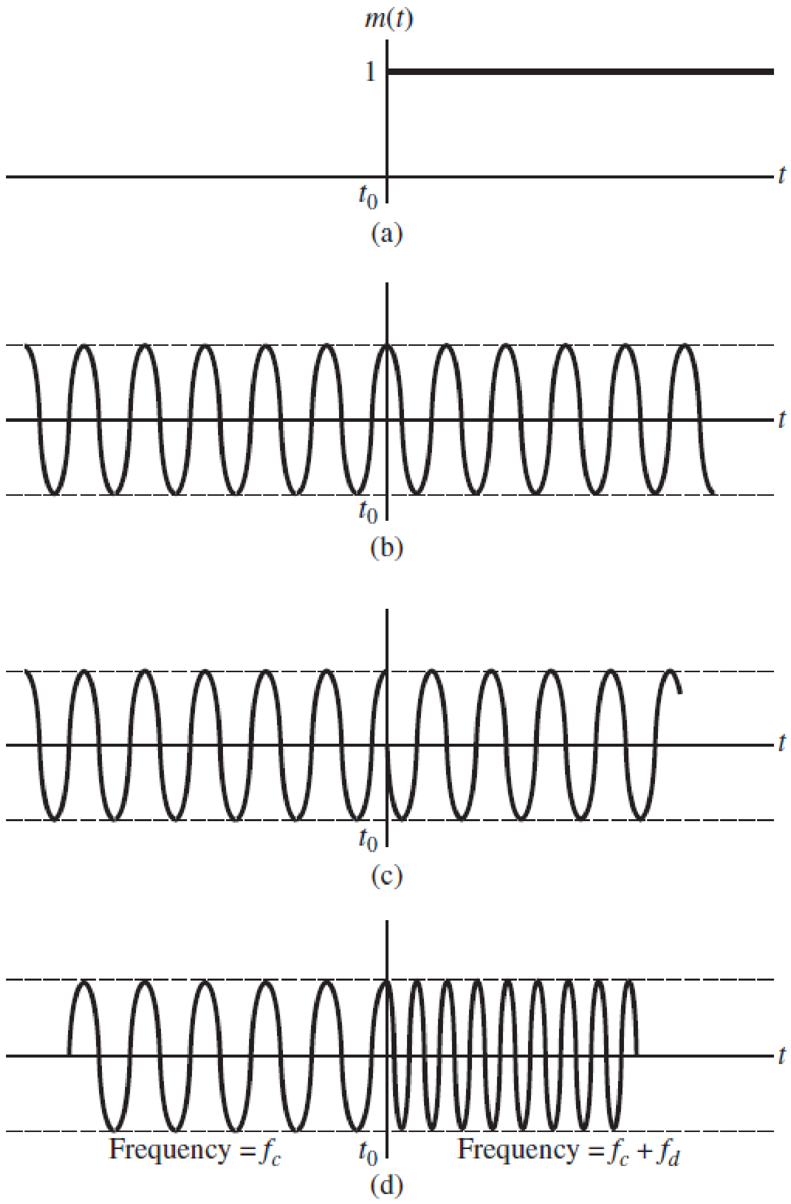
**Office Hours:**

BKD, 6th floor of Sirindhralai building

| | |
|---|---|
| Wednesday | 14:00-15:30 |
| Friday | 14:00-15:30 |

# Example 5.5: FM vs. PM

Figure 31




$m(t)$

Carrier: $A\cos(2\pi f_c t + \phi)$

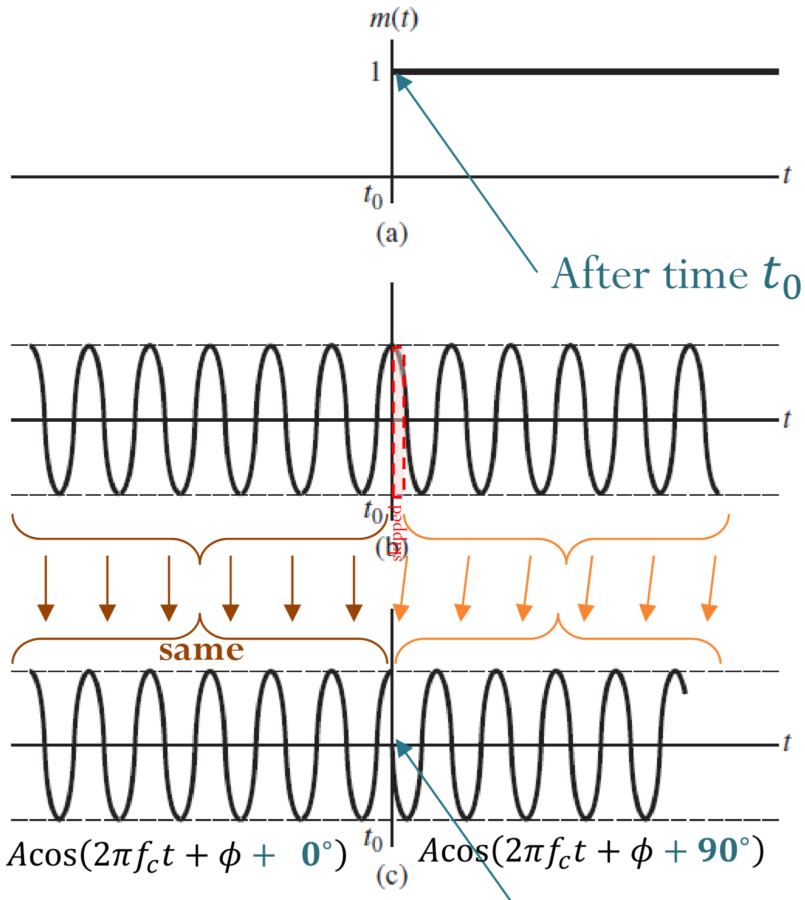
$x_{\mathrm{PM}}(t)$

$x_{\mathrm{FM}}(t)$

$$x_{PM}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

# Phase Modulation

Figure 31




$$m(t) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

After time $t_0$, the message jumps to the value 1.

$$A\cos(2\pi f_c t + \phi)$$

Here the phase modulator uses $k_p = 90°$

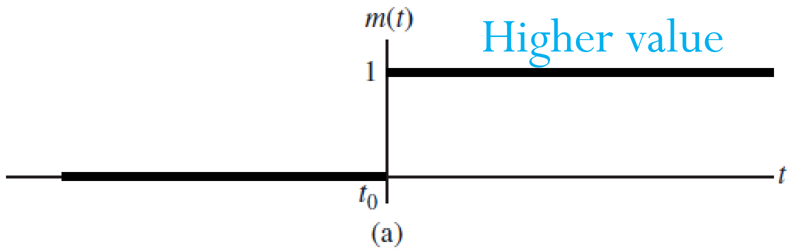
$$x_{PM}(t) = \begin{cases} A\cos(2\pi f_c t + \phi + \mathbf{0}°), & t < t_0 \\ A\cos(2\pi f_c t + \phi + \mathbf{90}°), & t > t_0 \end{cases}$$

After time $t_0$, the phase is skipped ahead (advanced) by **90°**.

$f(t) = f_c + k_f m(t)$

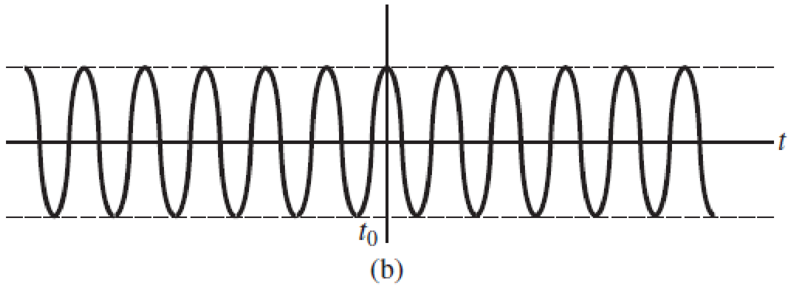
# Frequency Modulation

Figure 31




Higher value

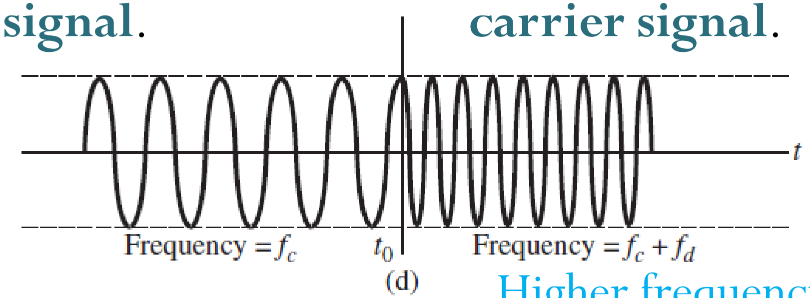
$m(t)$




$A\cos(2\pi f_c t + \phi)$

When the message $\boldsymbol{m(t)}$ is 0, the $\boldsymbol{x_{\text{FM}}(t)}$ has the **same frequency** as **the carrier signal**.

When the message $\boldsymbol{m(t)}$ is 1, the **frequency** of $\boldsymbol{x_{\text{FM}}(t)}$ is higher than the frequency of **the carrier signal**.

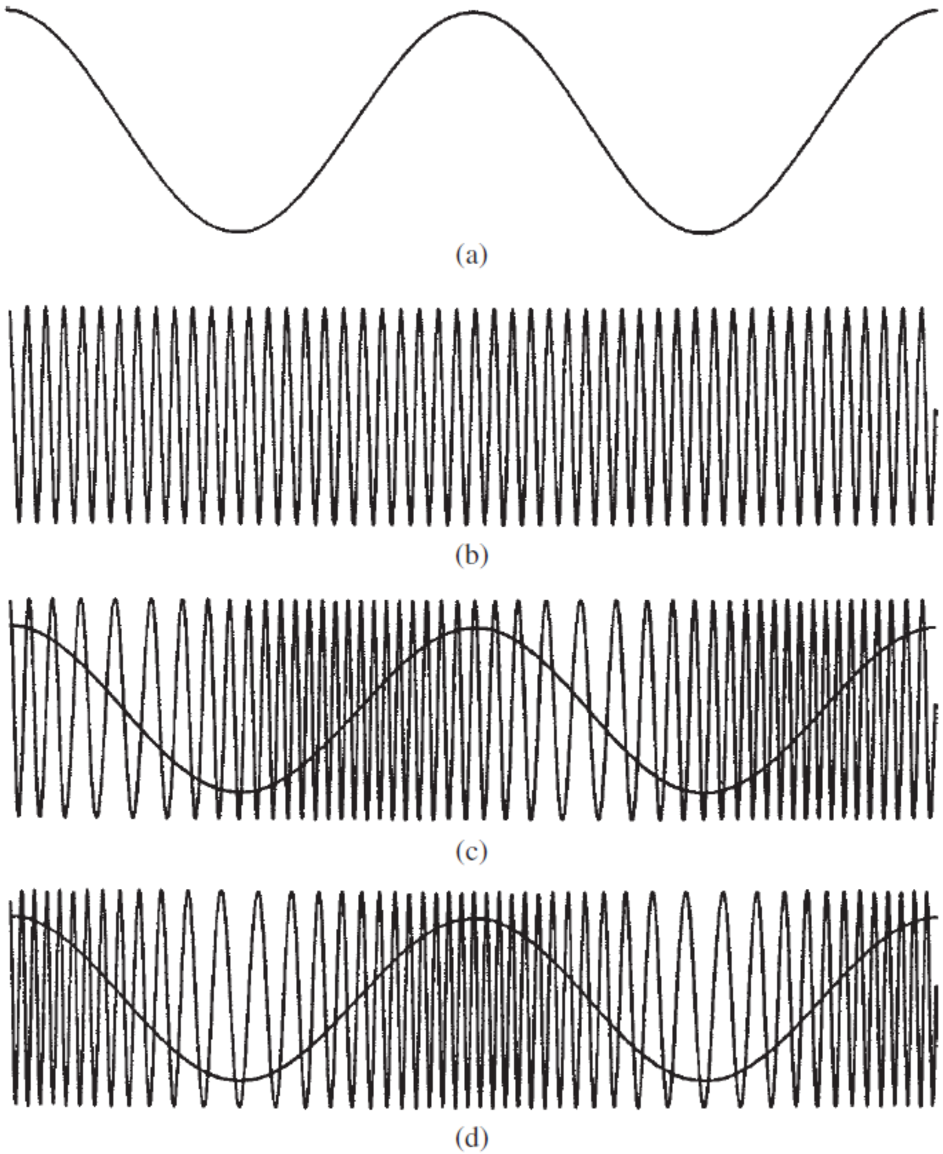



Higher frequency

$x_{\text{FM}}(t)$

# Example 5.6: FM vs. PM

Figure 32




$m(t)$

$A\cos(2\pi f_c t + \phi)$

$x_{\mathrm{PM}}(t)$

$x_{\mathrm{FM}}(t)$

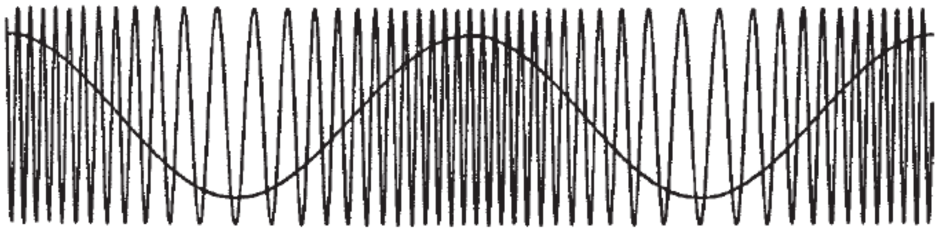
$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 32


(a)

It should be evident that the frequency is changing.

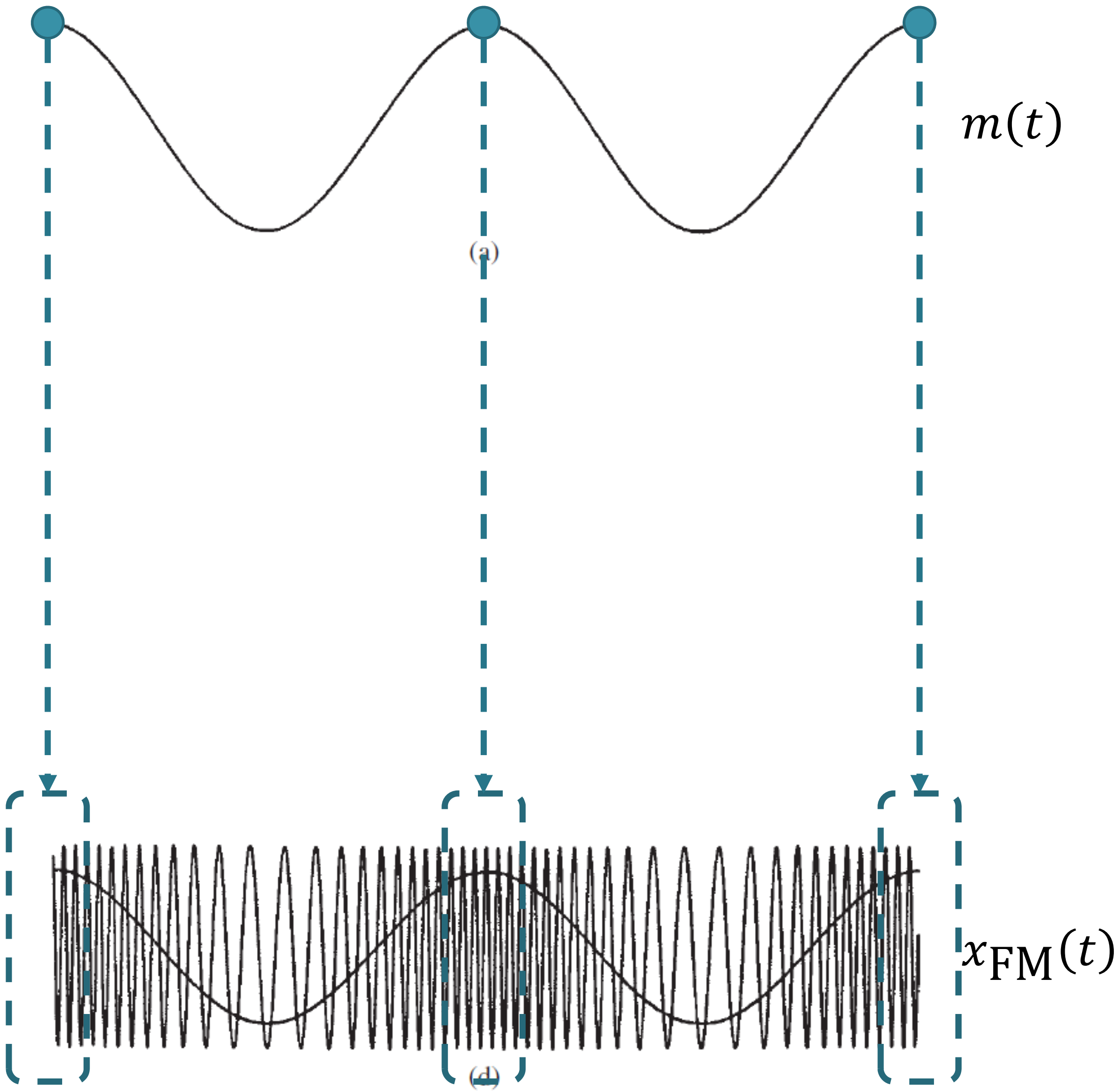



(d)

$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 32

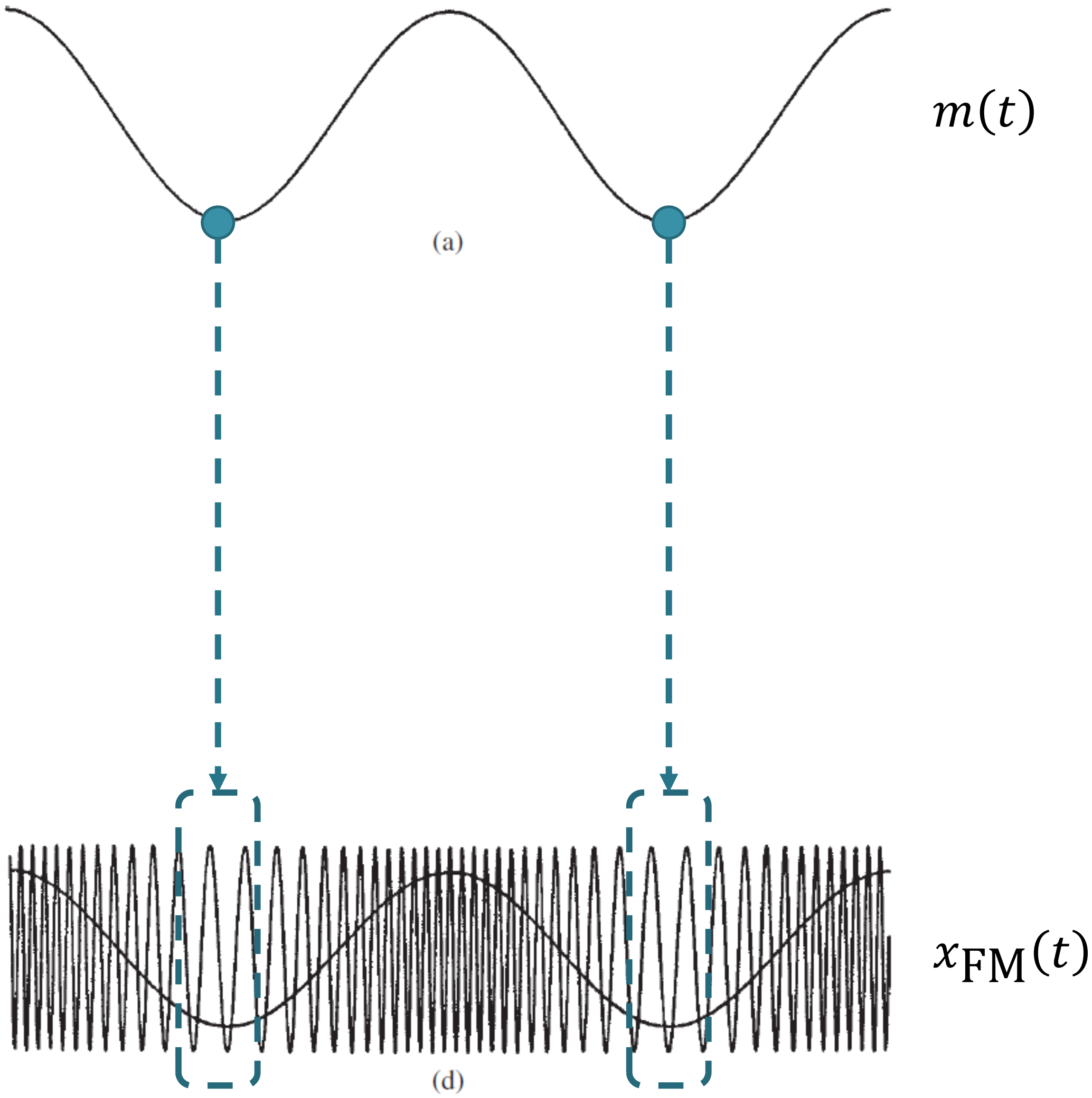



The time at which **$m(t)$** is at its **maximum** value corresponds to the time at which **$x_{\text{FM}}(t)$** has **maximum frequency**.

$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 32

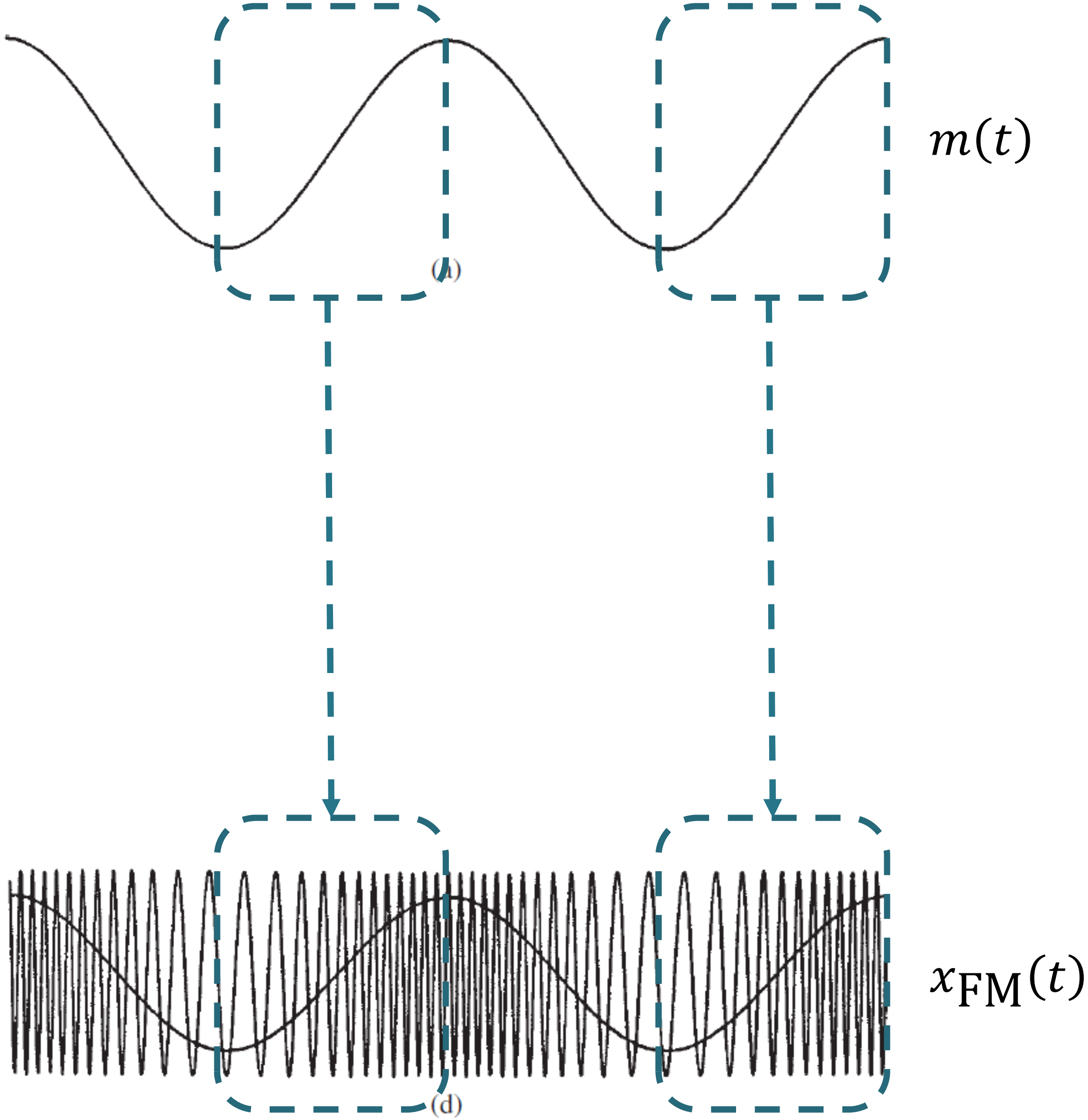



The time at which **$m(t)$** is at its **minimum** value corresponds to the time at which **$x_{FM}(t)$** has **minimum frequency**.

$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 32

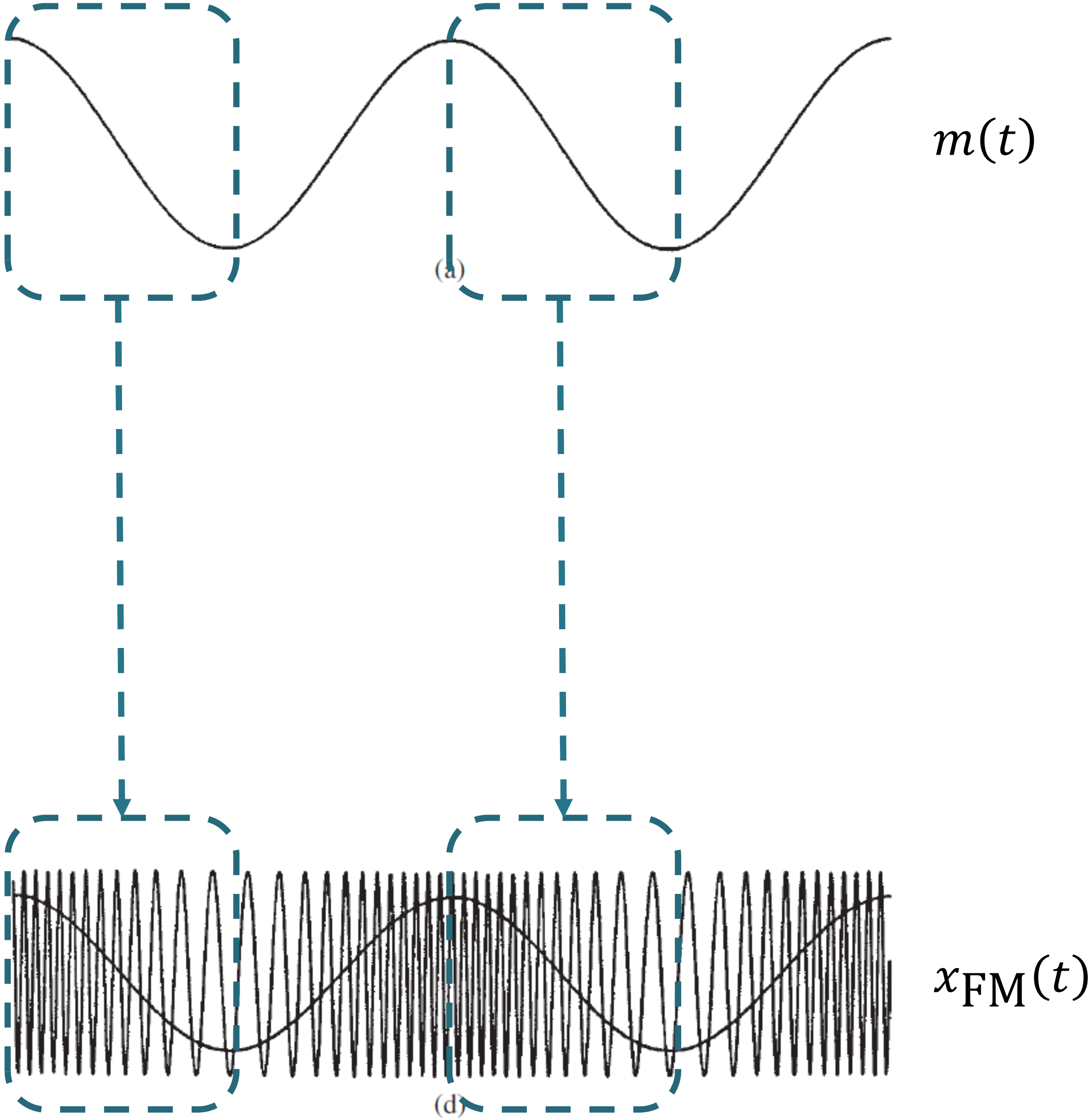



The time interval during which $\boldsymbol{m(t)}$ is **increasing** corresponds to the time interval during which $\boldsymbol{x_{FM}(t)}$ has **increasing frequency**.

$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 32




The time interval during which $\boldsymbol{m(t)}$ is **decreasing** corresponds to the time interval during which $\boldsymbol{x_{\text{FM}}(t)}$ has **decreasing frequency**.

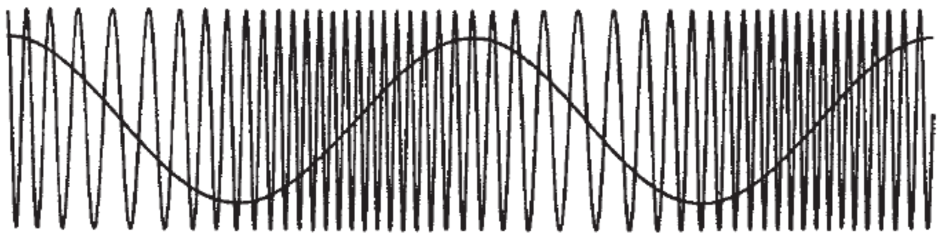
$$x_{\mathrm{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

# Phase Modulation

Figure 32


(a)

In $x_{\mathrm{PM}}(t)$, the **phase** varies in proportion with $m(t)$.
When $m(t)$ and hence the phase of $x_{\mathrm{PM}}(t)$ change **continuously**, it is difficult to see the connection with the actual plot of $x_{\mathrm{PM}}(t)$.




(c)

$$x_{\mathrm{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

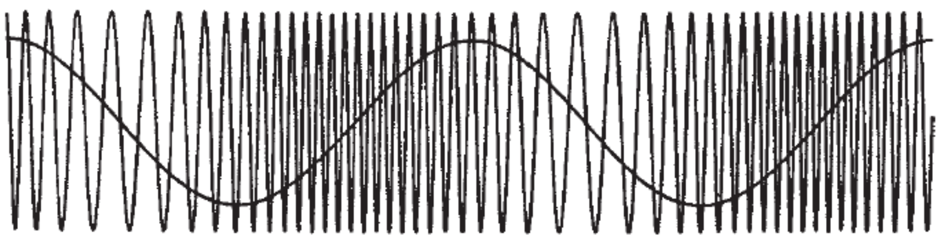
# Phase Modulation

Figure 32

$m(t)$

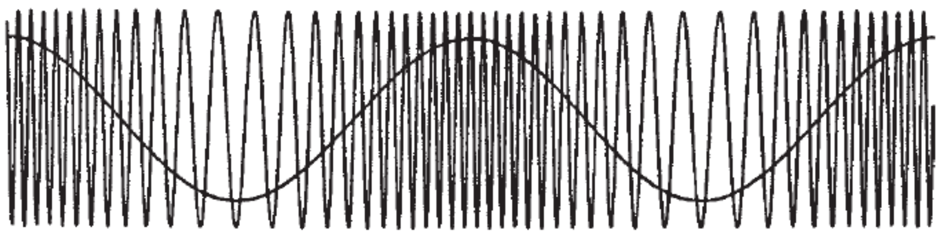
(a)

One may notice here that, in this example, $x_{\mathrm{PM}}(t)$ is similar to $x_{\mathrm{FM}}(t)$ Except that the graph is shifted. However, it is still not clear (visually) how the graph of $x_{\mathrm{PM}}(t)$ is related to $m(t)$.



$x_{\mathrm{PM}}(t)$

(c)

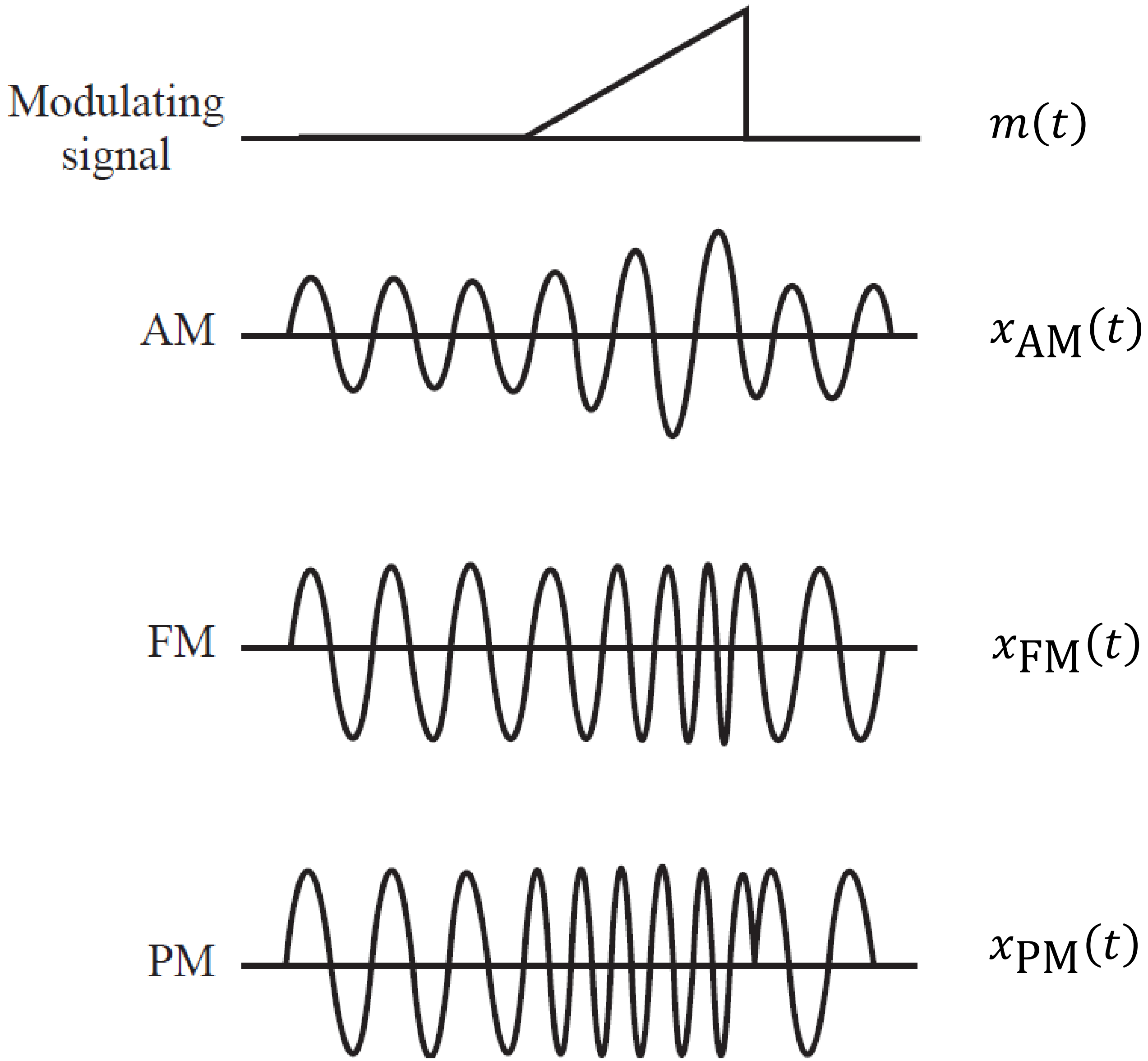


$x_{\mathrm{FM}}(t)$

(d)

# AM, FM, and PM

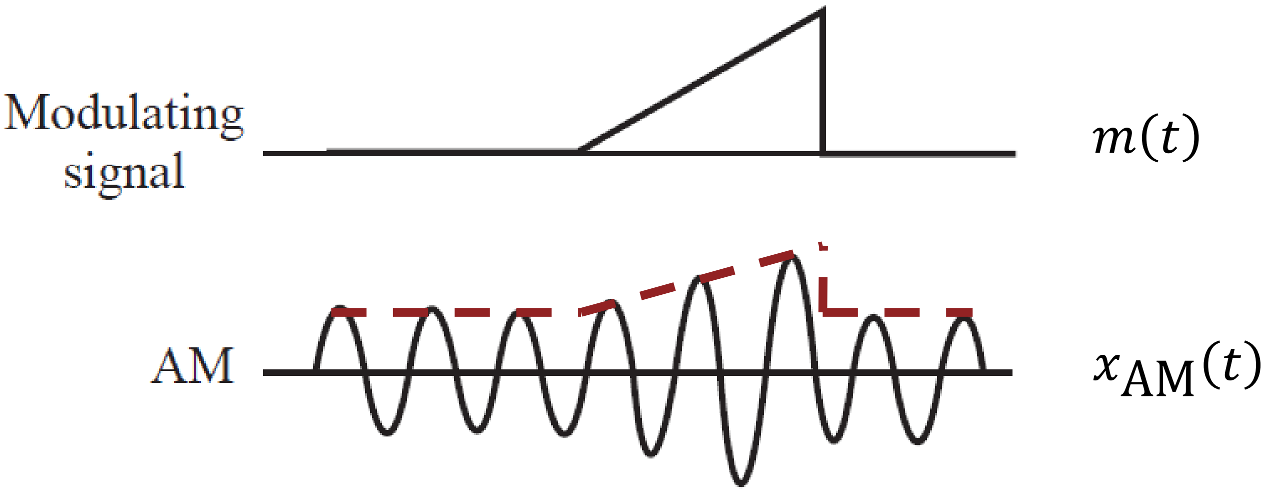
Figure 33



$$x_{AM}(t) = \big(A + m(t)\big)\cos(2\pi f_c t + \phi)$$

# Amplitude Modulation

Figure 33

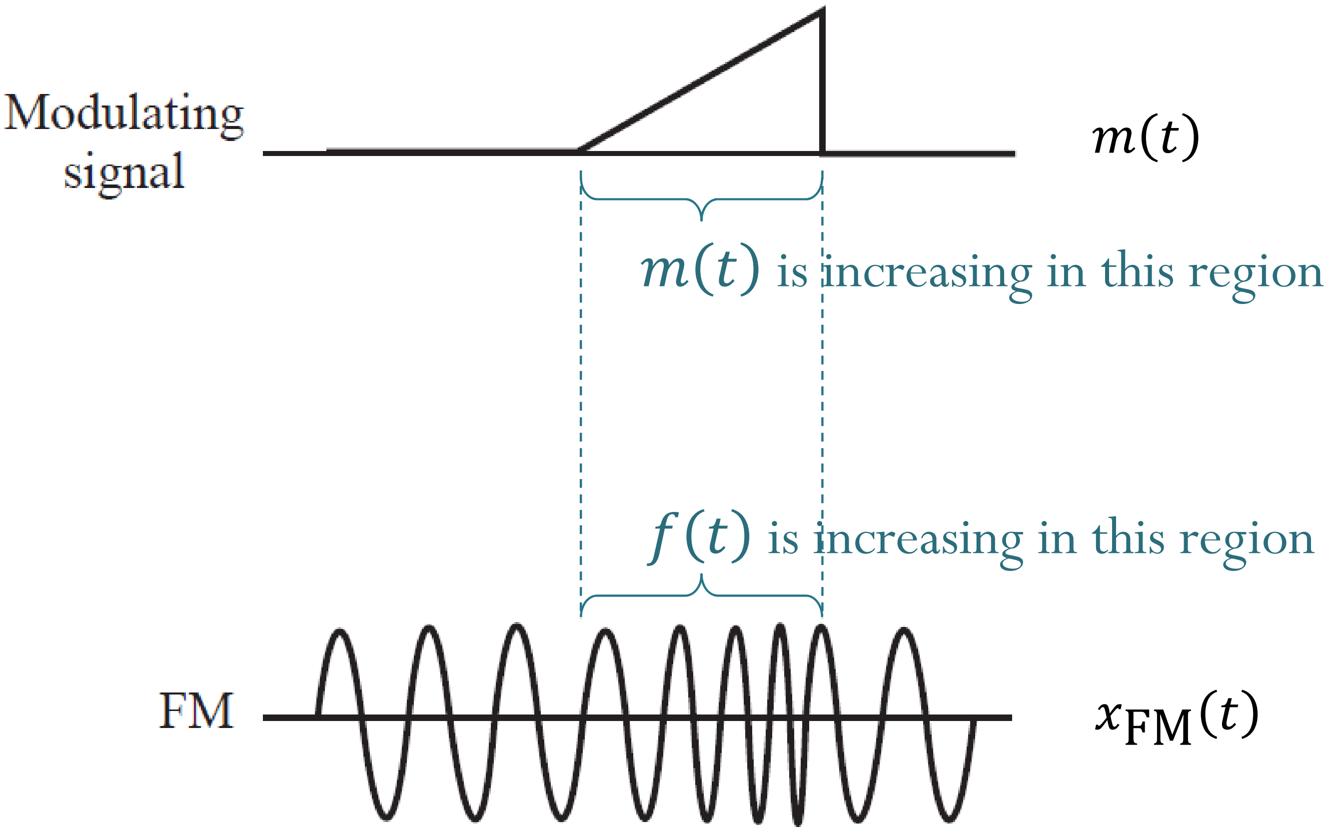



In $x_{AM}(t)$, the **envelope** varies in proportion to $A + m(t)$.

$f(t) = f_c + k_f m(t)$

# Frequency Modulation

Figure 33

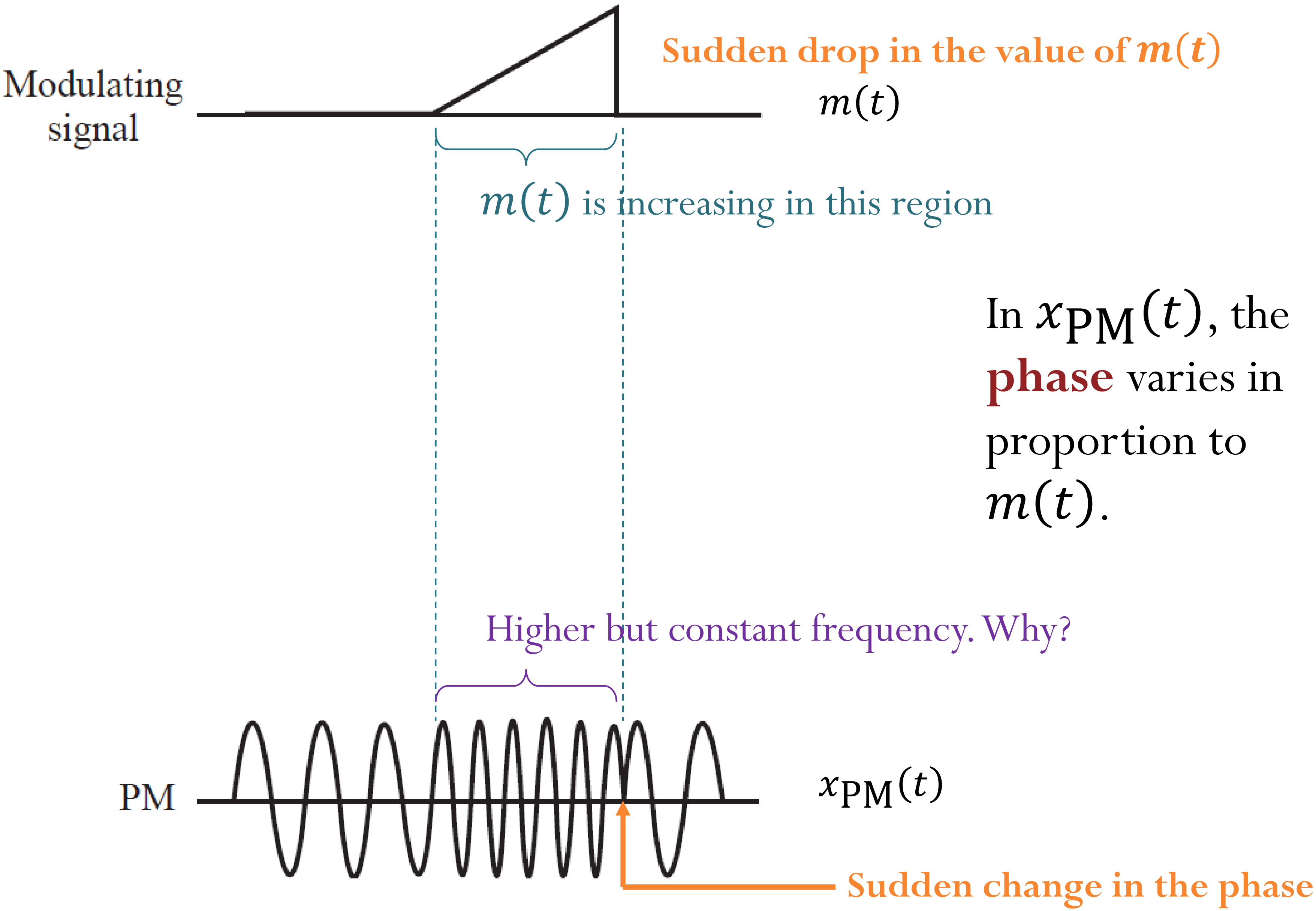



In $x_{\mathrm{FM}}(t)$, the **frequency** varies in proportion to $m(t)$.

$x_{PM}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$

# Phase Modulation

Figure 33




In $x_{PM}(t)$, the **phase** varies in proportion to $m(t)$.

# Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

  - Frequency = $\boldsymbol{f_0}$
- Generalized sinusoidal signal:

$$g(t) = A\cos\big(\theta(t)\big)$$

  - Frequency = ?
    - Observation: Frequency value may vary as a function of time.
      - "**instantaneous frequency**"
- Why do we need to find the instantaneous frequency?
  - Analyze Doppler effect (or Doppler shift)
  - Implement **frequency modulation** (**FM**)
    - where the instantaneous frequency will follow the message $m(t)$.

# Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

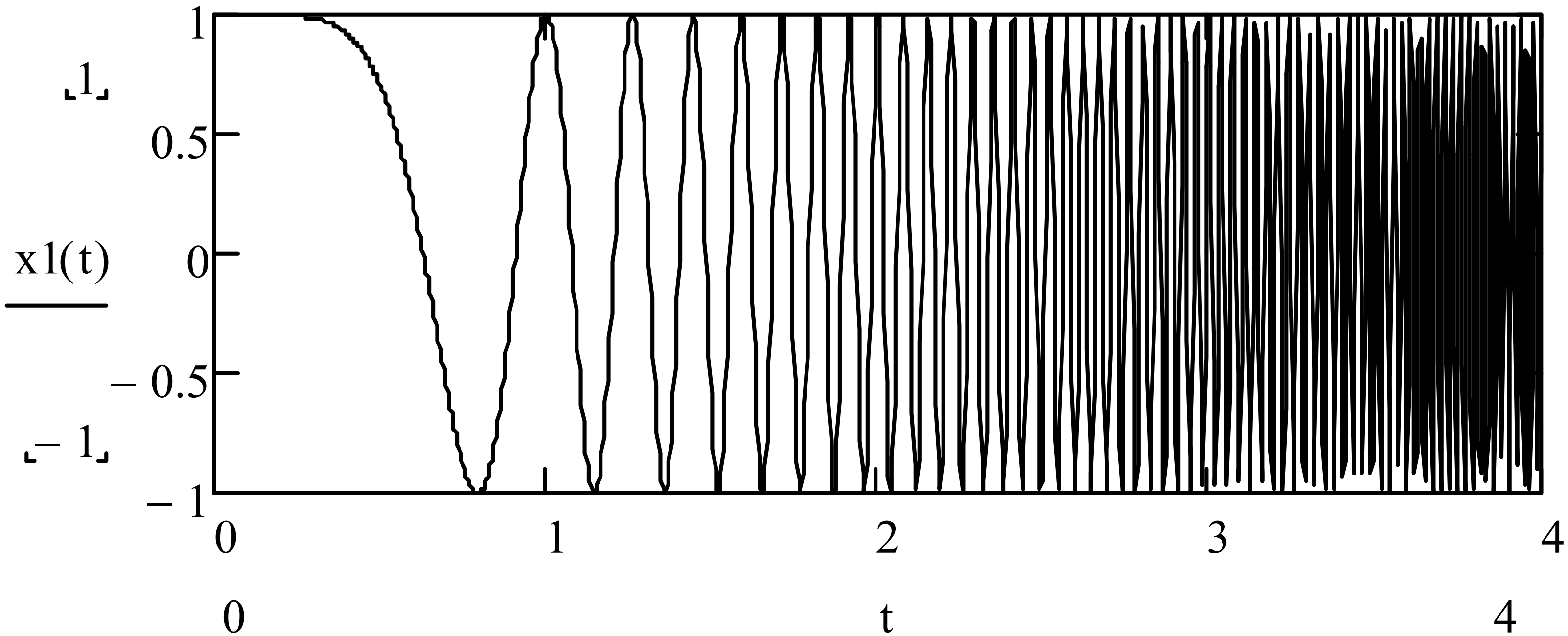



Figure 35

At t = 2, frequency = ?

# Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

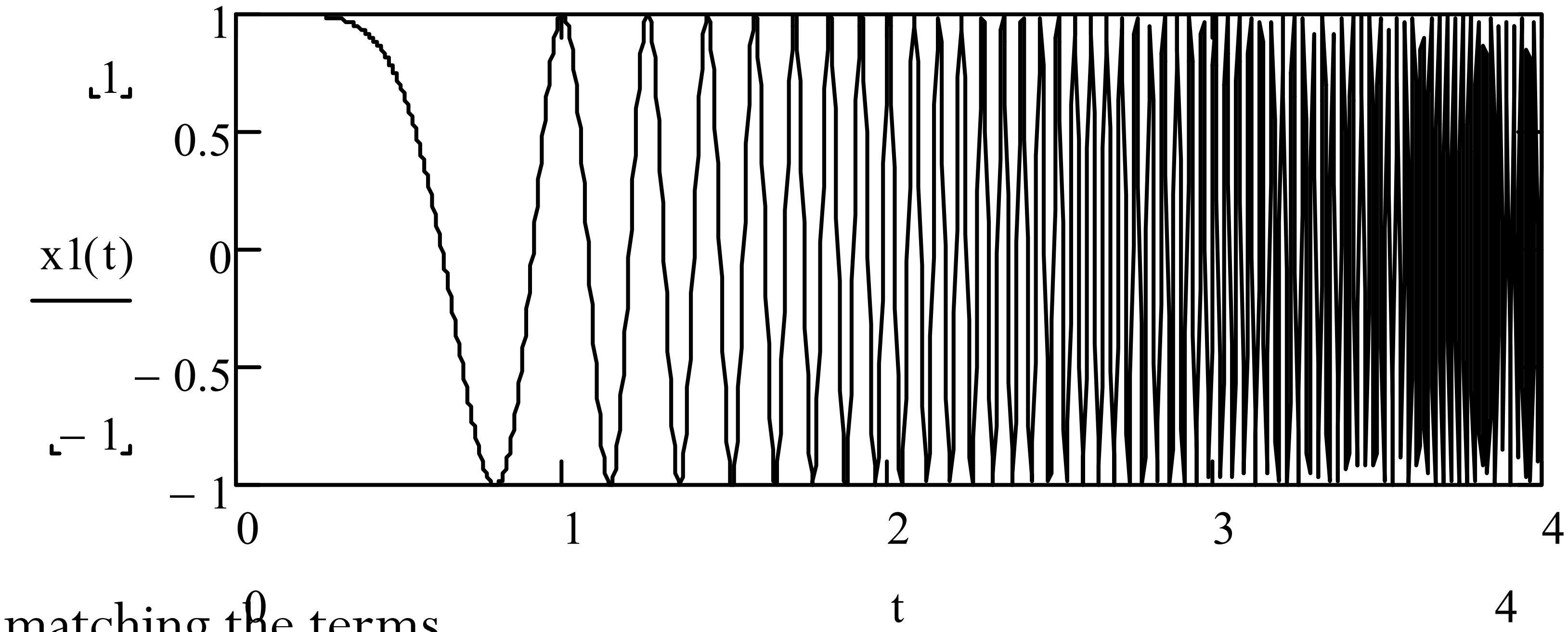



Figure 35

By matching the terms with $\cos(2\pi \boldsymbol{f}_0 t)$, you may guess that $\boldsymbol{f}(\boldsymbol{t}) = t^2$.

At t = 2, $f = t^2 = 4$ Hz?

Correct?

# Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

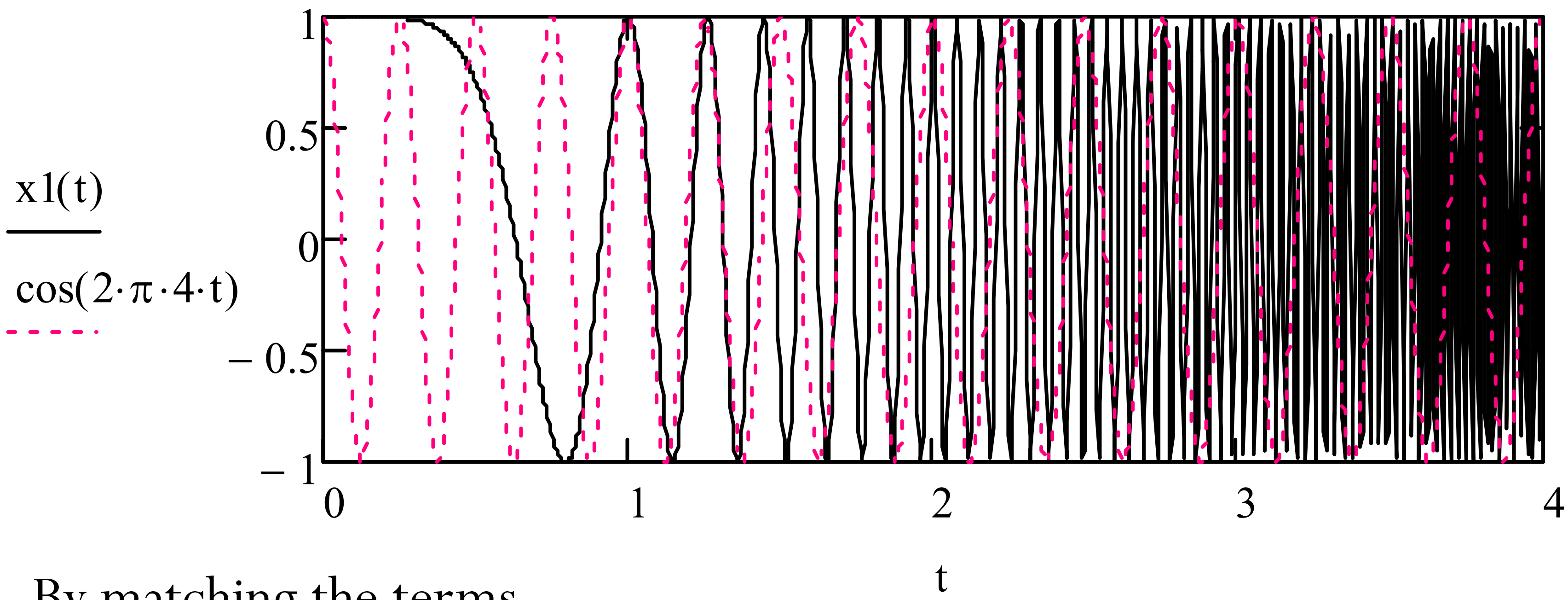



Figure 35

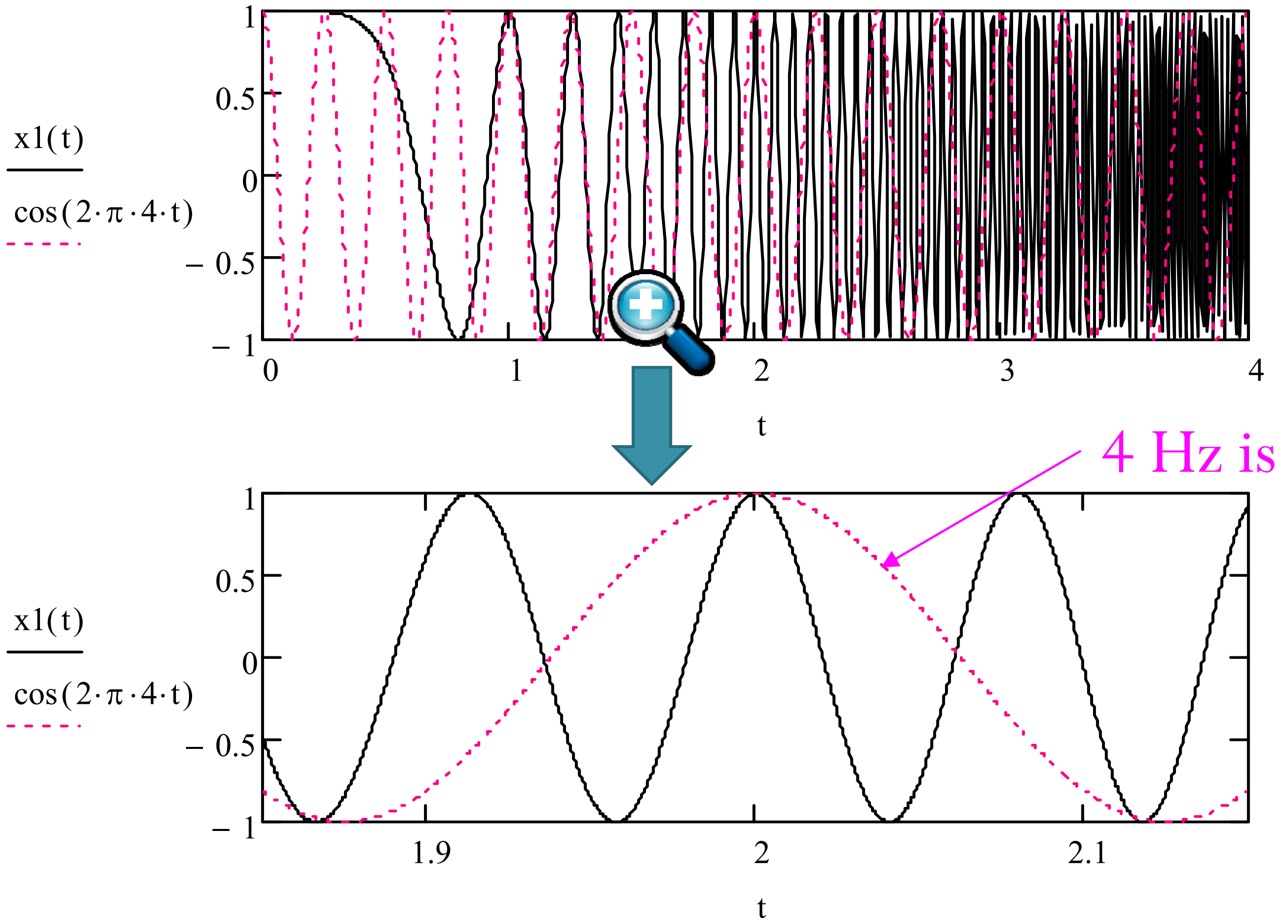
By matching the terms with $\cos(2\pi f_0 t)$, you may guess that $f(t) = t^2$.

At t = 2, $f = t^2 = 4$ Hz?

Correct?

# Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$




4 Hz is too low!!!

Figure 35

# Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

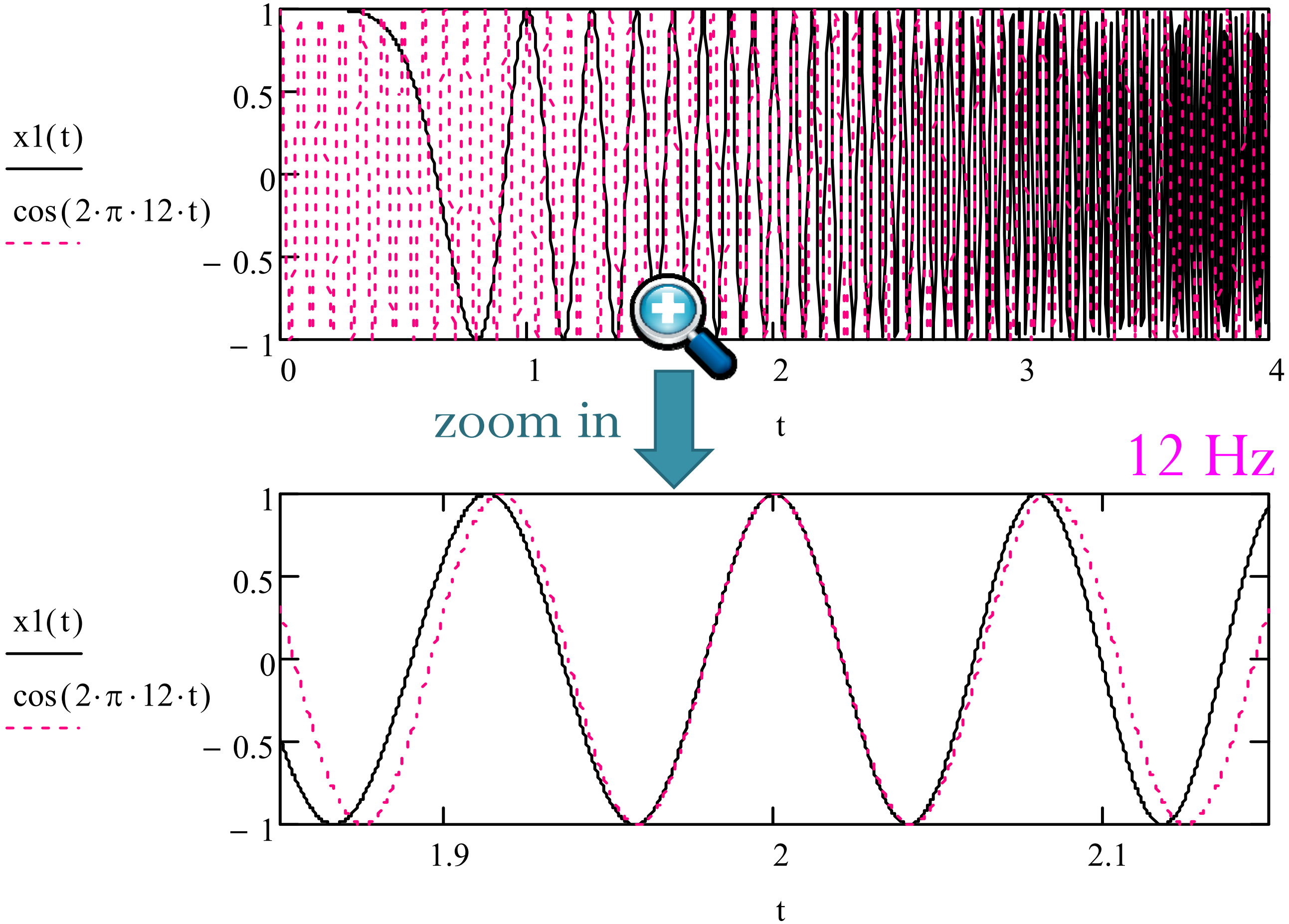


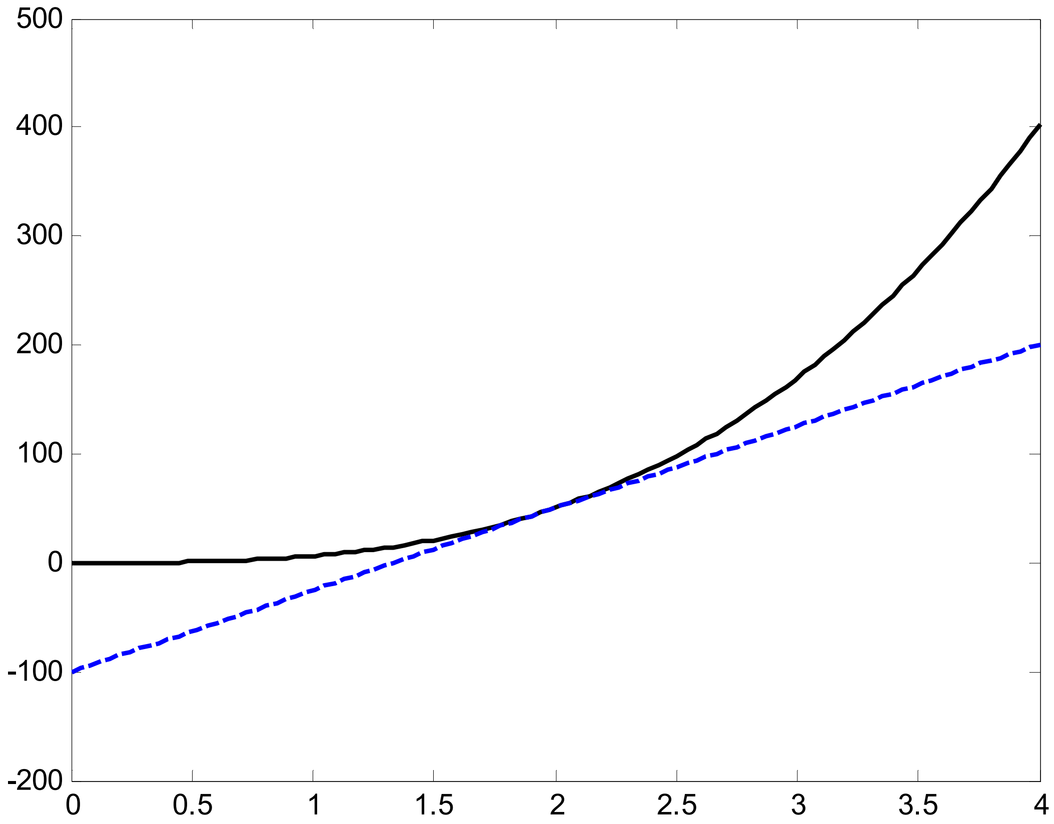

Figure 35

12 Hz fits better.

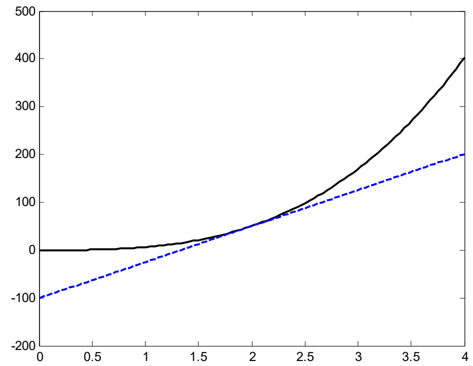
12 Hz?

# First-order (straight-line) approximation/linearization

- How does the formula $f(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization



# First-order (straight-line) approximation/linearization




- How does the formula $f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization
  - When we consider a function $\theta(t)$ near a particular time, say, $t = t_0$, the value of the function is approximately

$$\theta(t) \approx \underbrace{\theta'(t_0)}_{\text{slope}}(t - t_0) + \theta(t_0) = \underbrace{\theta'(t_0)}_{\text{slope}}t + \underbrace{\theta(t_0) - t_0\theta'(t_0)}_{\text{constant}}$$

  - Therefore, near $t = t_0$,

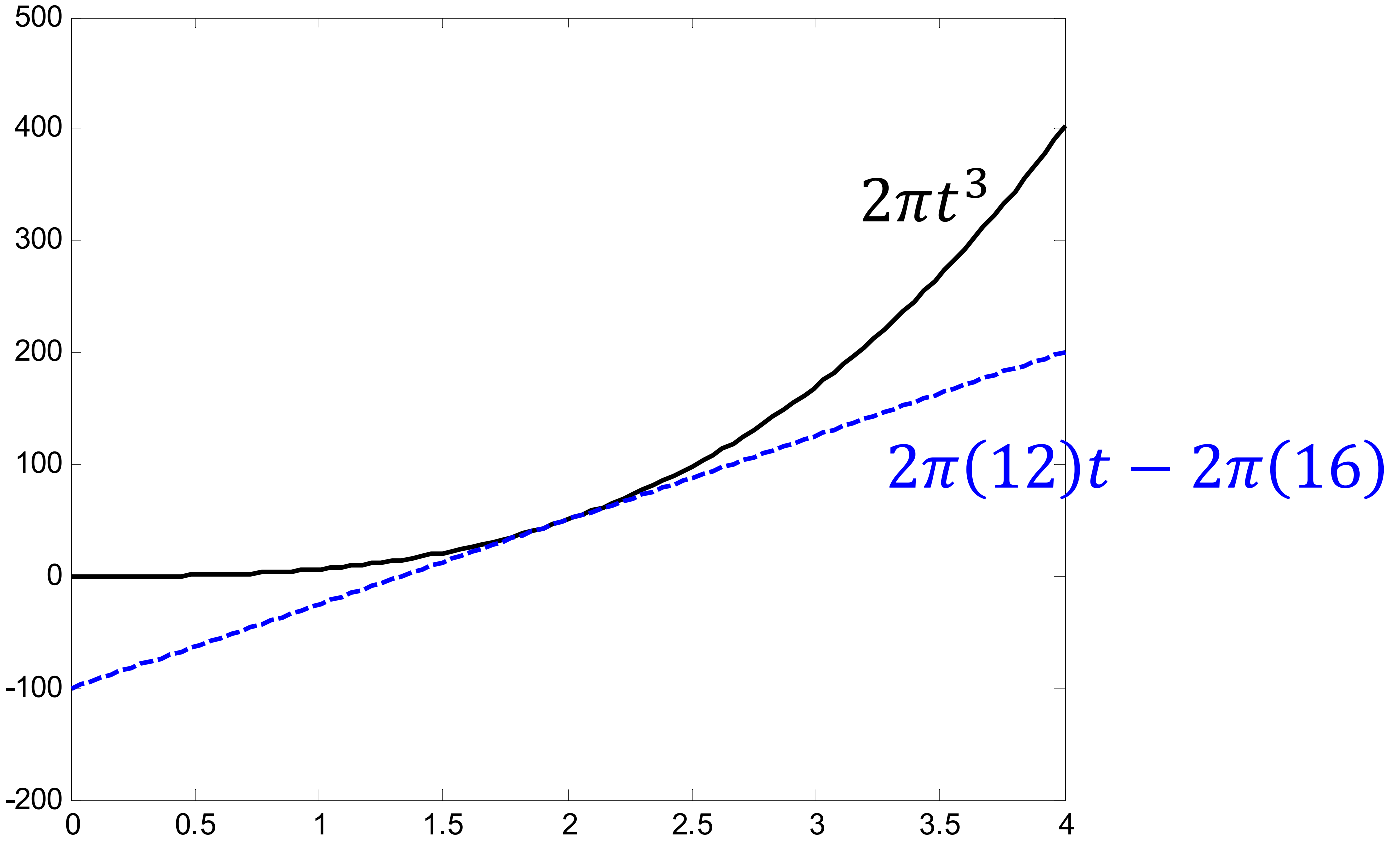
$$\cos(\theta(t)) \approx \cos(\theta'(t_0)t + \theta(t_0) - t_0\theta'(t_0))$$

  - Now, we can directly compare the terms with $\cos(2\pi f_0 t + \phi)$.

# First-order (straight-line) approximation/linearization

- For example, for $t$ near $t = 2$,

$$2\pi t^3 \approx 2\pi\left(3t^2\right)\Big|_{t=2}(t-2)+2\pi t^3\Big|_{t=2} = 2\pi(12)t-2\pi(16)$$



# First-order (straight-line) approximation/linearization

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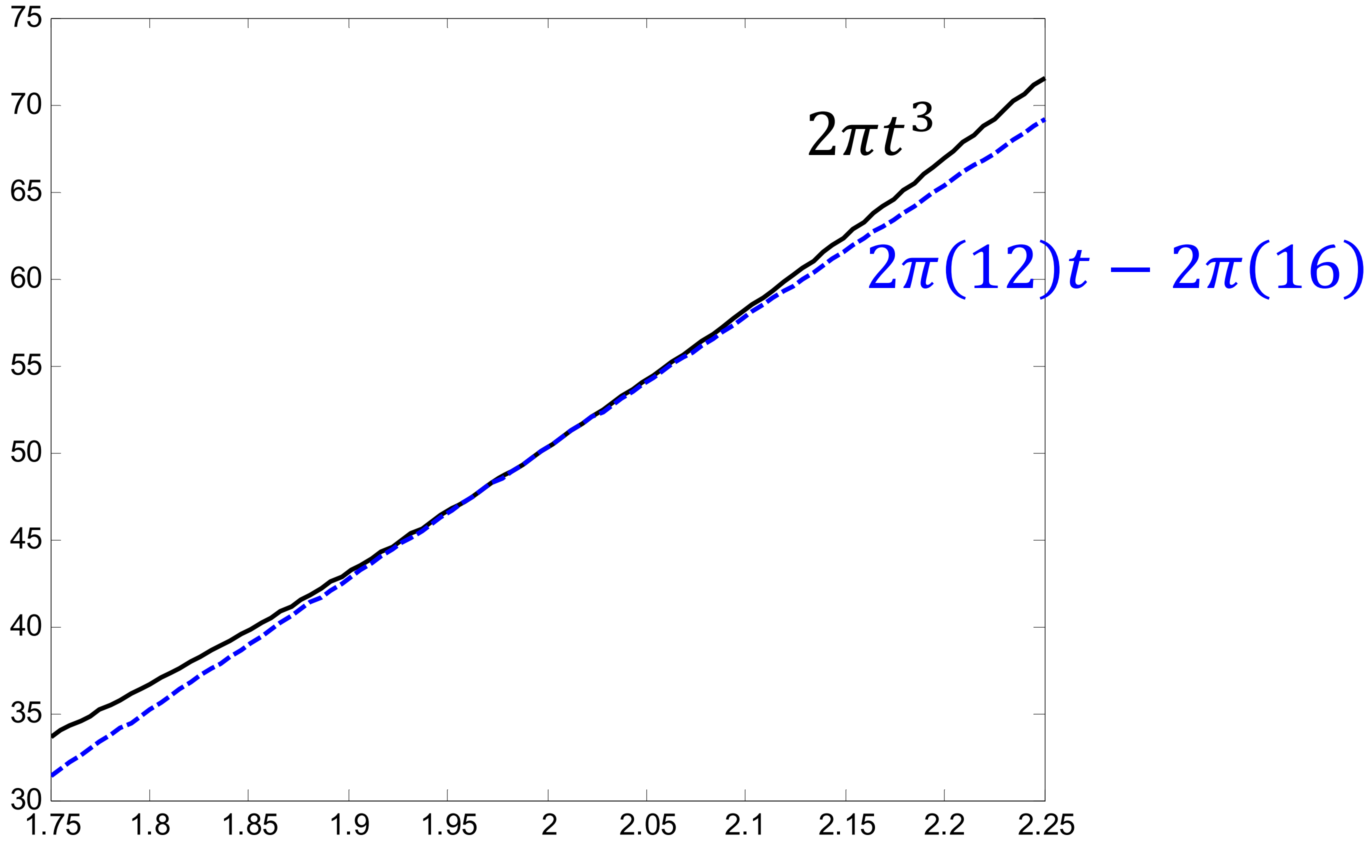

# First-order (straight-line) approximation/linearization

- For example, for $t$ near $t = 2$,

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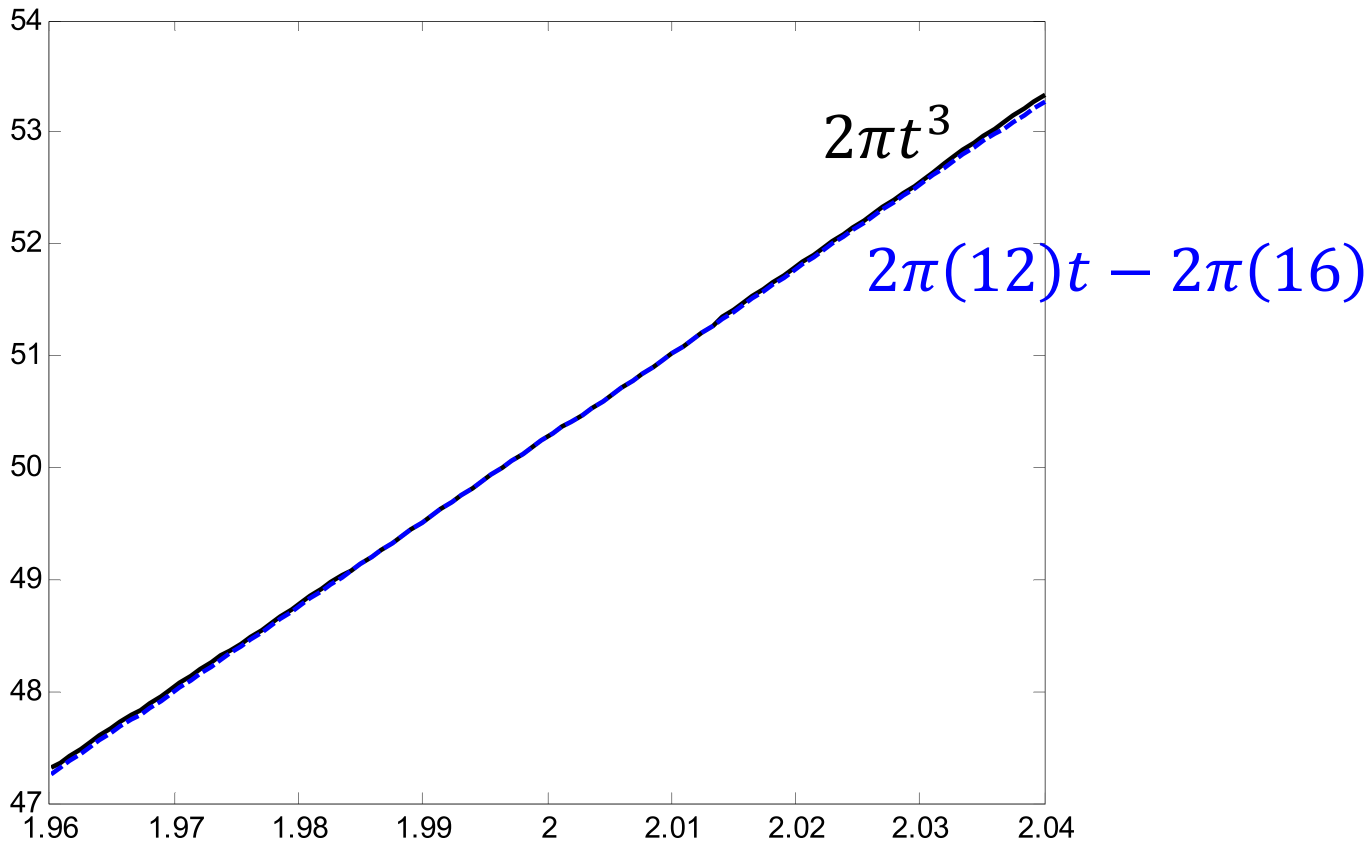


# Same idea

- Suppose we want to find $\sqrt{15.9}$.
- Let $g(x) = \sqrt{x}$.
  - Note that $\frac{d}{dx} g(x) = \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.
- Approximation: $g(x) \approx g'(x_0)(x - x_0) + g(x_0)$
- 15.9 is near 16.
- $\sqrt{15.9} = g(15.9)$
  - $\approx g'(16)(15.9 - 16) + g(16)$
  - $= \frac{1}{2\sqrt{16}}(-0.1) + \sqrt{16} = -\frac{0.1}{8} + 4 = 3.9875$
- MATLAB:

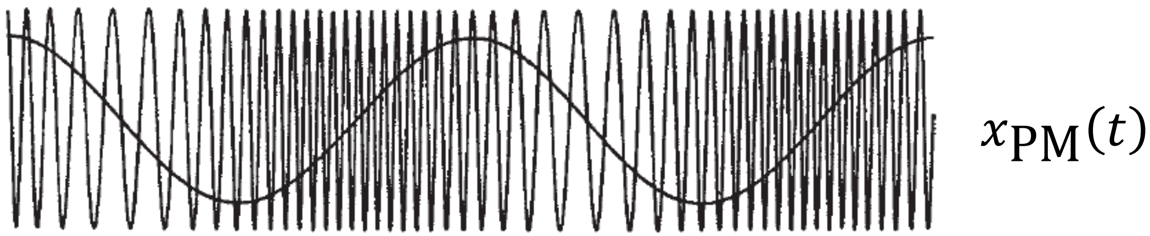
```
>> sqrt(15.9)
ans =
    3.987480407475377
```

# Phase Modulation

Figure 32


(a)

When $m(t)$ and hence the phase of $x_{\mathrm{PM}}(t)$ change **continuously**, it is difficult to see the connection with the actual plot of $x_{\mathrm{PM}}(t)$.

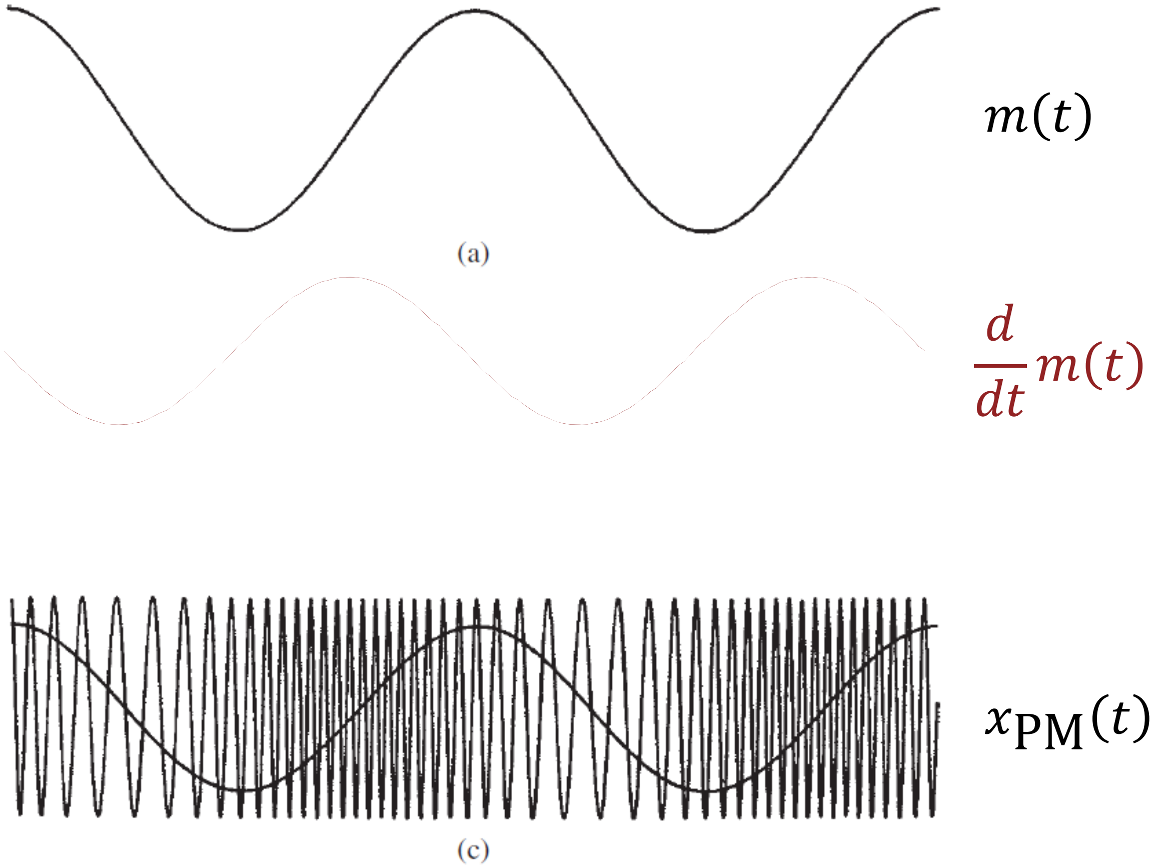



(c)

New Fact: In $x_{\mathrm{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

# Phase Modulation

Figure 36

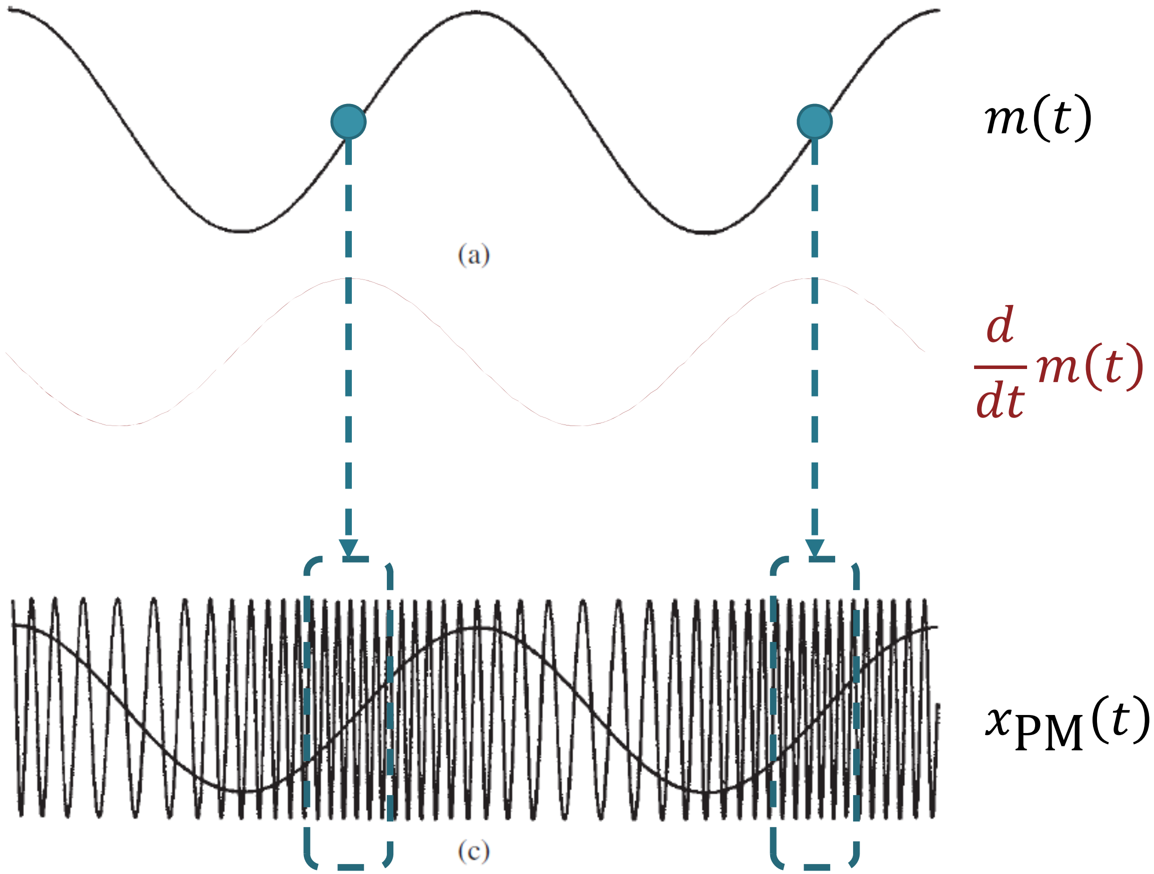



$$f(t) = f_c + k_p \frac{d}{dt} m(t)$$

New Fact: In $x_{\text{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

# Phase Modulation

Figure 36

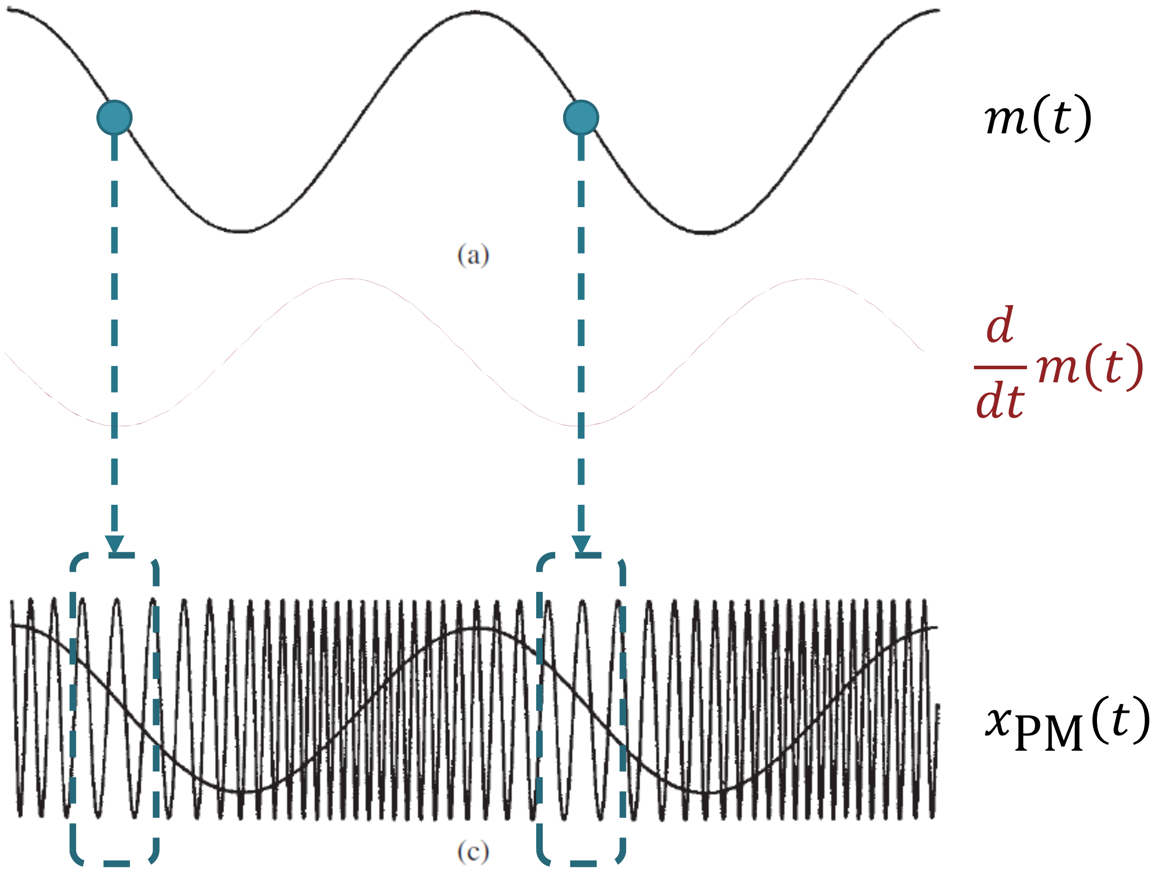



The time at which the **slope** of $\boldsymbol{m(t)}$ is at its **maximum** value corresponds to the time at which $\boldsymbol{x_{\mathrm{PM}}(t)}$ has **maximum frequency**.

New Fact: In $x_{\mathrm{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

# Phase Modulation

Figure 36

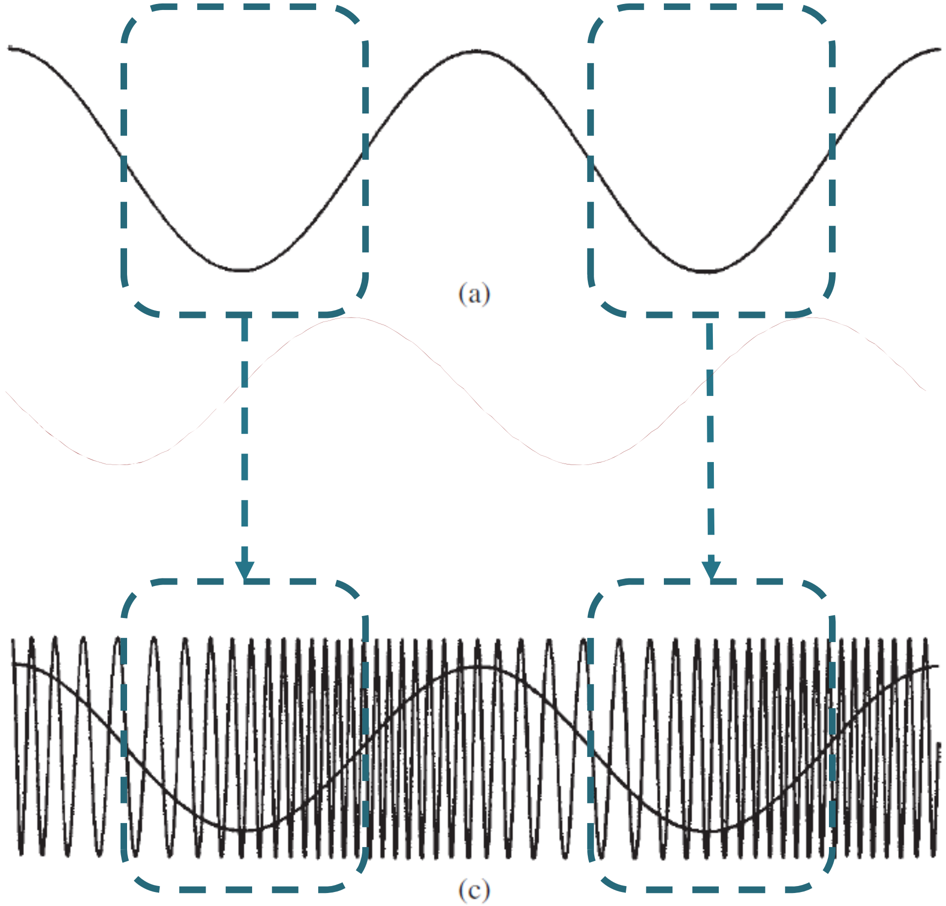



The time at which the **slope** of $\boldsymbol{m(t)}$ is at its **minimum** value corresponds to the time at which $\boldsymbol{x_{\mathrm{PM}}(t)}$ has **minimum frequency**.

New Fact: In $x_{\mathrm{PM}}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

# Phase Modulation

Figure 36




$m(t)$

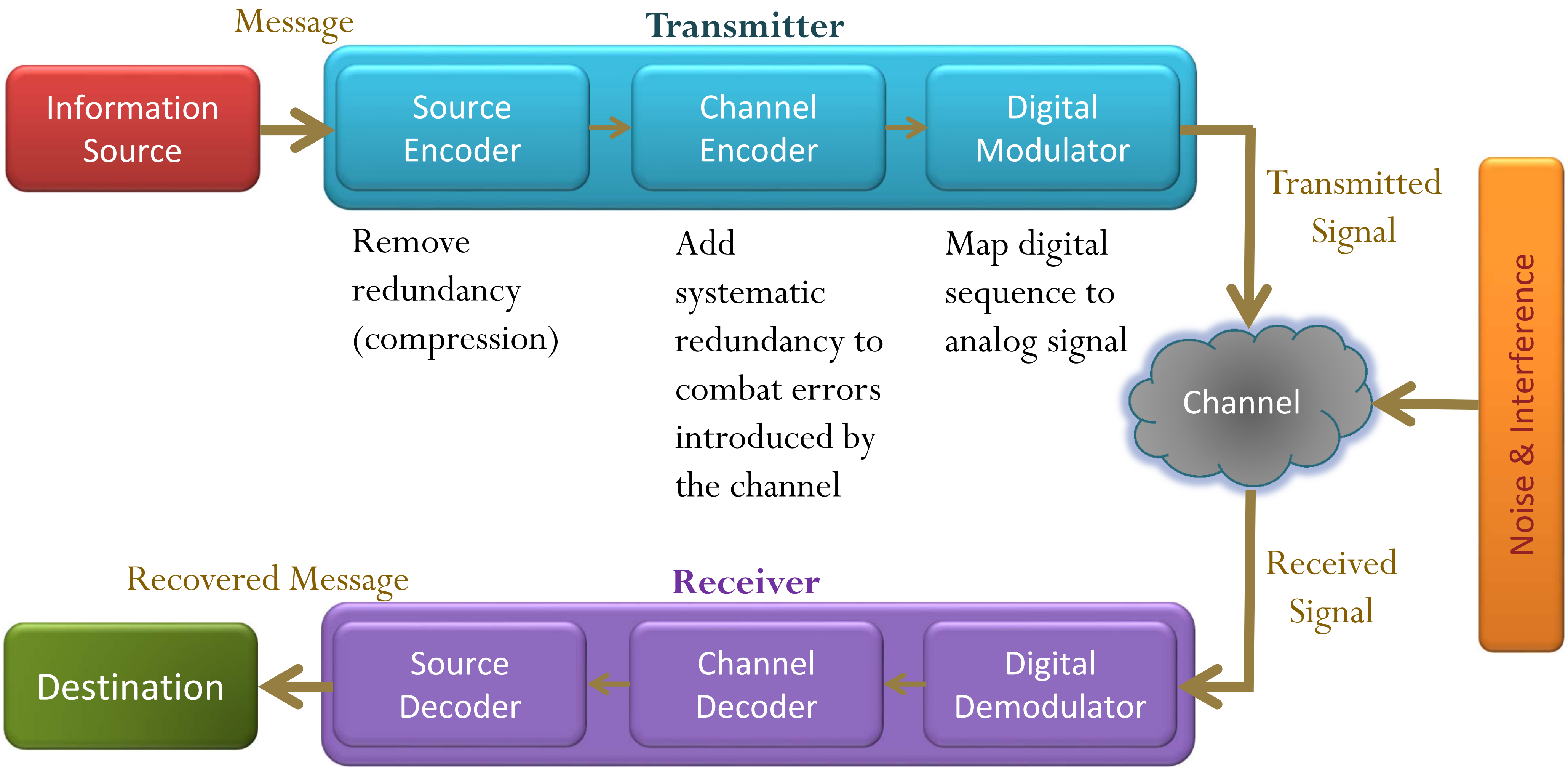
$\frac{d}{dt}m(t)$

$x_{PM}(t)$

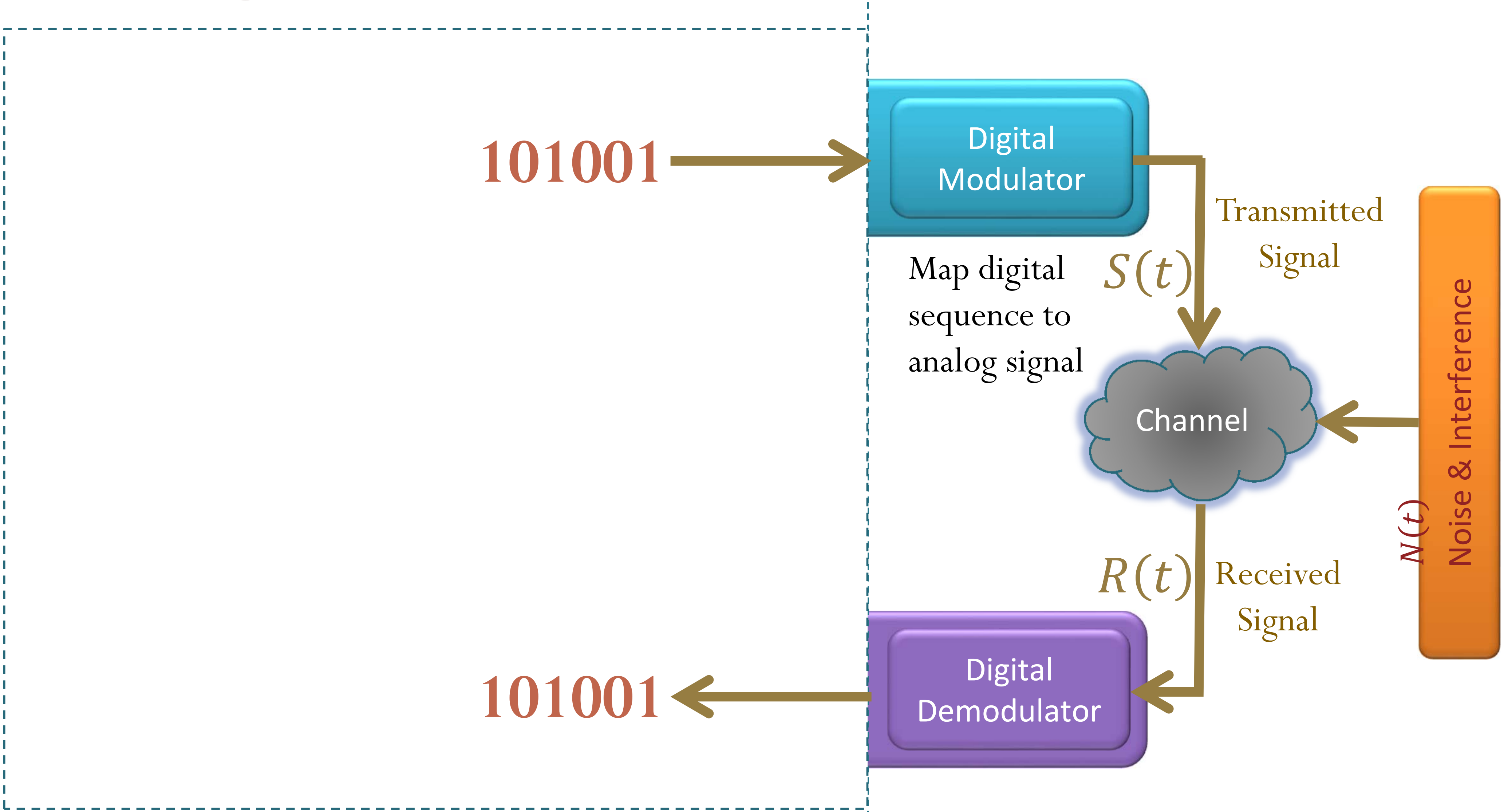
The time interval during which the slope of $m(t)$ is increasing corresponds to the time interval during which $x_{PM}(t)$ has increasing frequency.

New Fact: In $x_{PM}(t)$, the **instantaneous frequency** varies in proportion with the **slope** of $m(t)$.

# Elements of digital commu. sys.



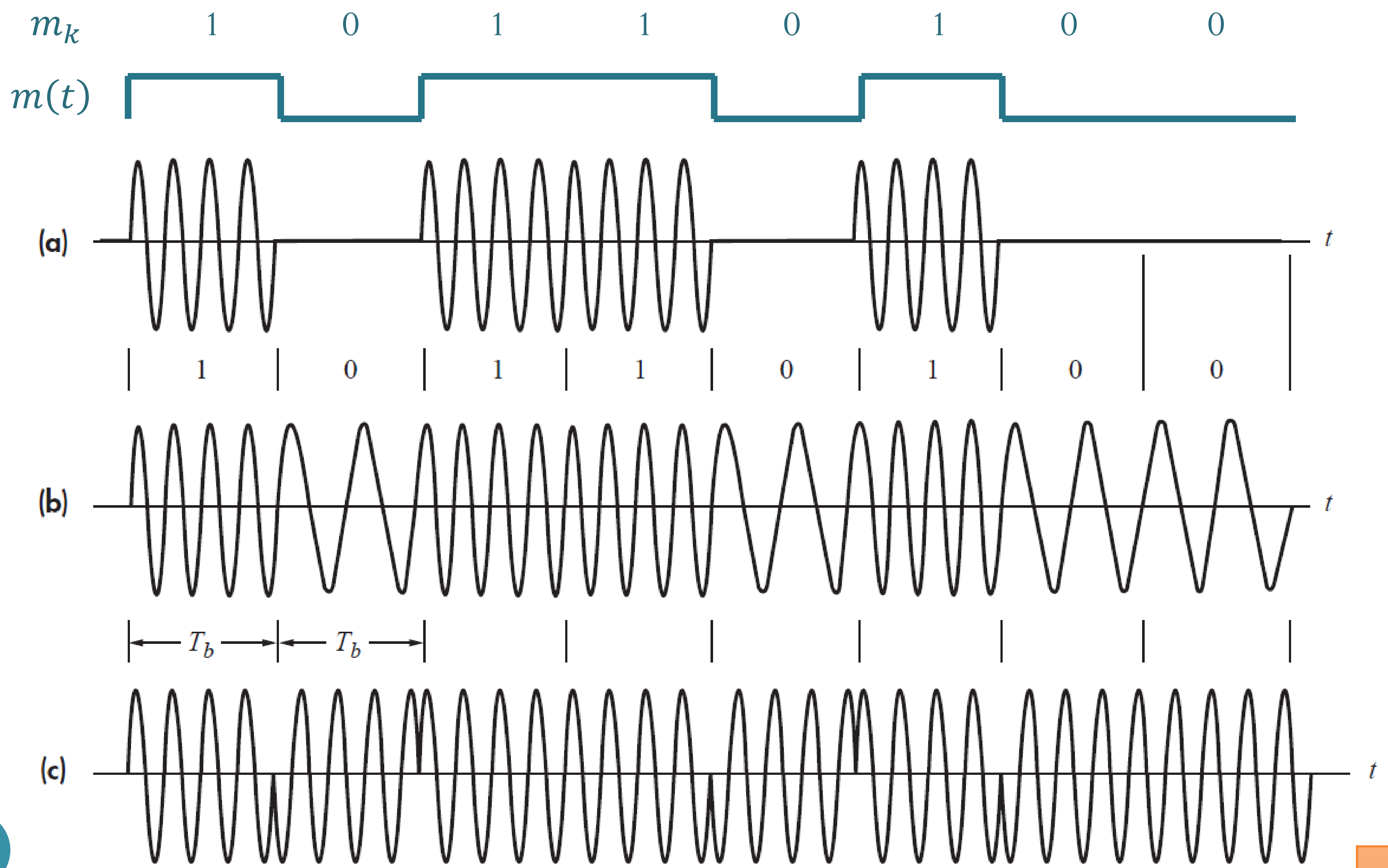
# Digital Modulation/Demodulation



# Digital Version

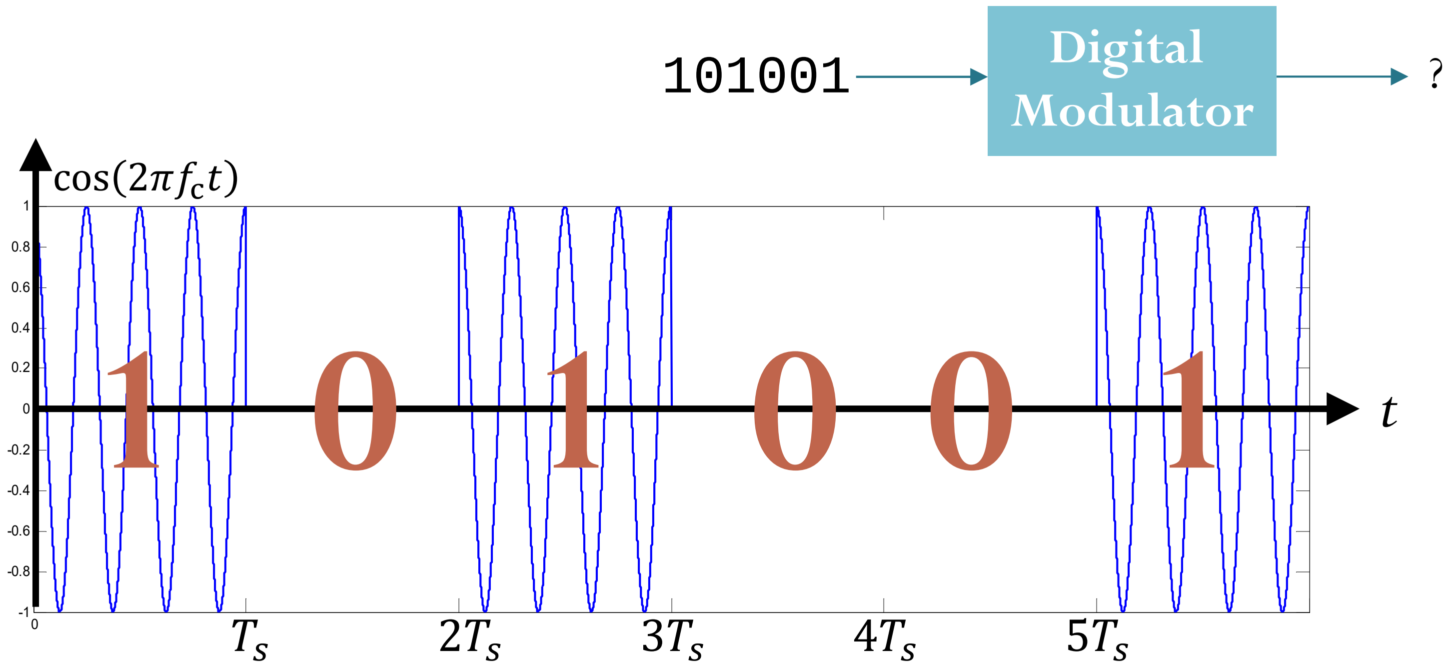
- Use digital signal to modulate the amplitude, frequency, or phase of a sinusoidal carrier wave.
  - Think of $m(t)$ as a train of scaled (rectangular) pulses.
  - The modulated parameter will be switched or keyed from one discrete value to another.
- Three basic forms:
  - amplitude-shift keying (ASK)
  - frequency-shift keying (FSK)
  - phase-shift keying (PSK)

# Binary ASK, FSK, and PSK




[Carlson & Crilly, 2009, Fig 14.1-1 p 649]

# Simple ASK: ON-OFF Keying (OOK)




$f_c = 4$ Hz
Bit rate = 1 bps

$t$

[ASK_playTones_Demo.m]

# Simple "ASK": "ON-OFF Keying"

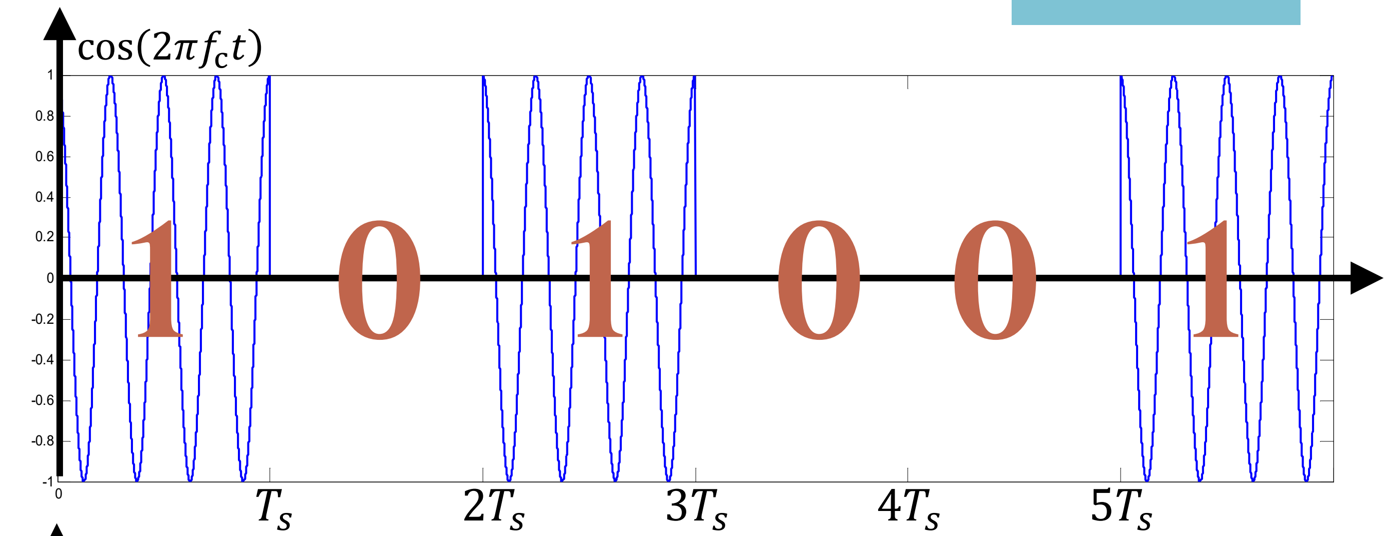
**Smoke signal**

"It's no use the signal's too weak."

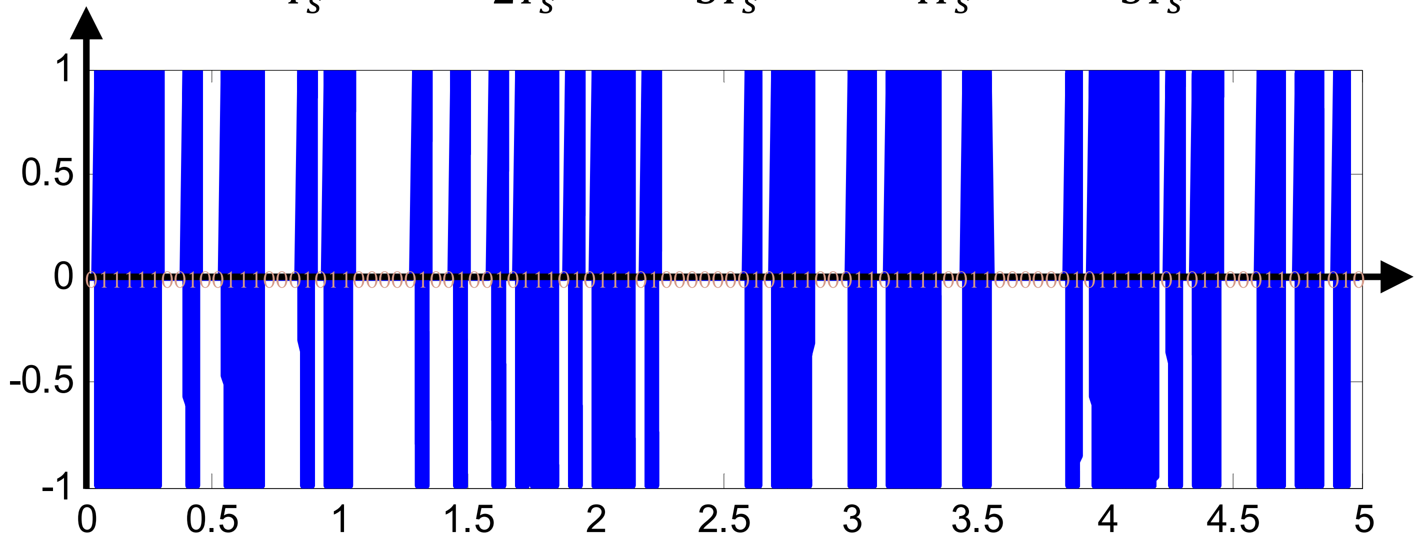
# Simple ASK: ON-OFF Keying (OOK)

Bit stream → Digital Modulator → ?

$f_c = 4$ Hz
Bit rate = 1 bps

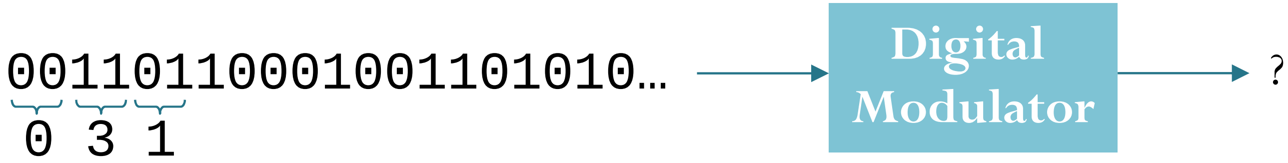


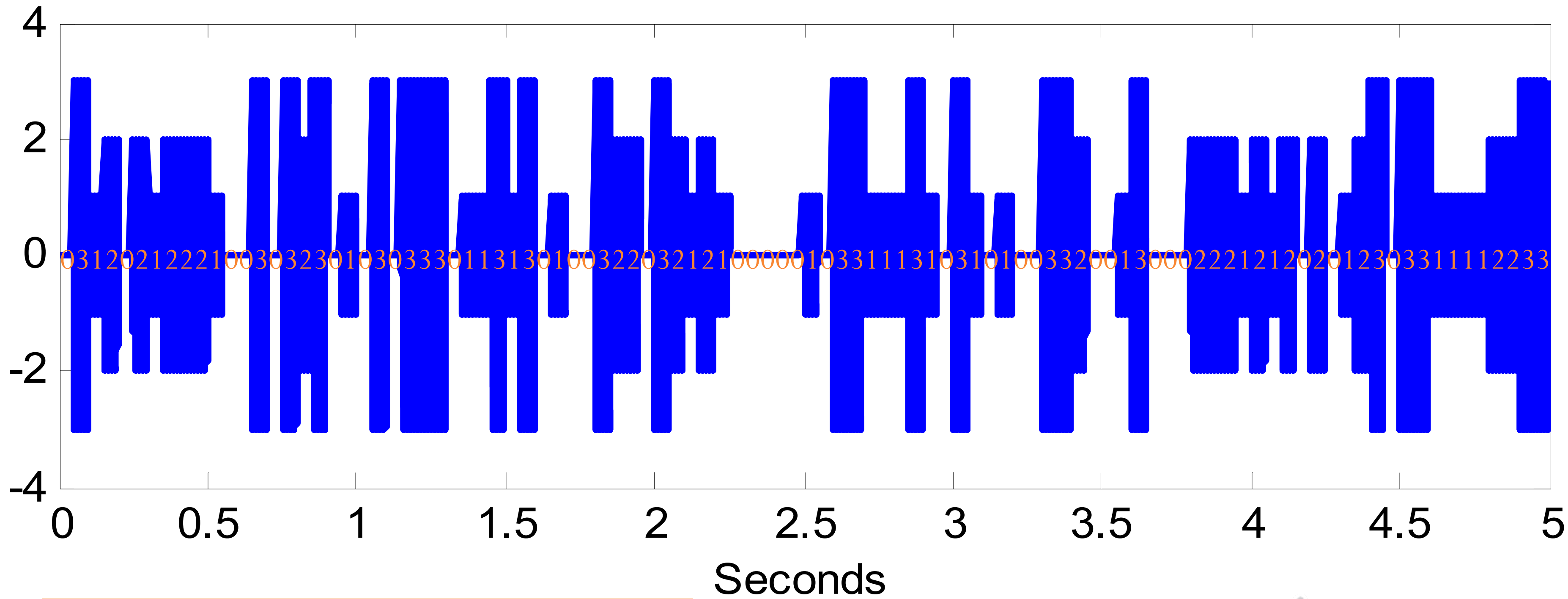

$f_c = 100$ Hz
Bit rate = 20 bps




[ASK_playTones_Demo.m]

# ASK: Higher Order Modulation



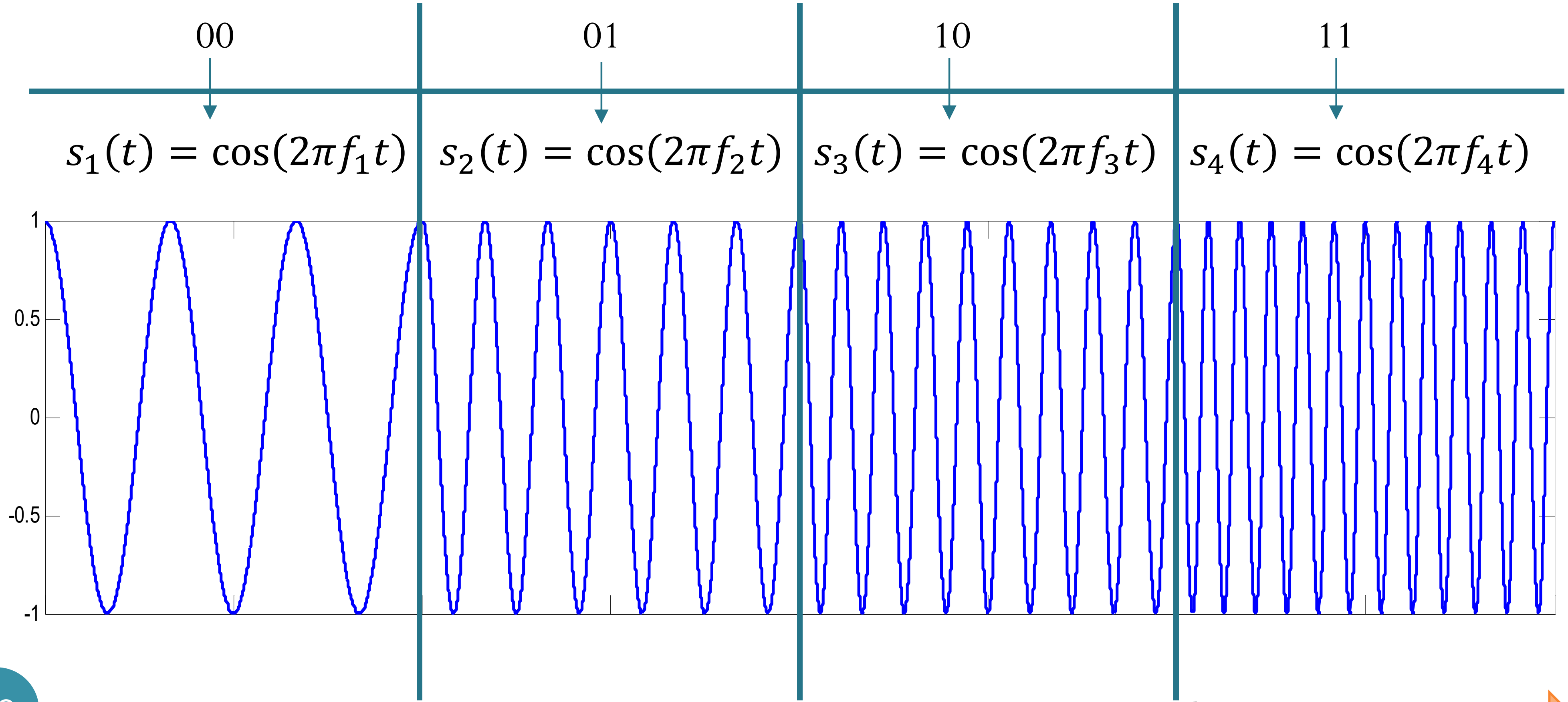

$f_c = 100$ Hz

Symbol rate = 20 symbols per second

Bit rate = 40 bps

# FSK

$M = 4$ $f_c \in \{f_1, f_2, f_3, f_4\} = \{3, 6, 9, 12\}$ [Hz]




[FSK_playTones_Demo.m]

# FSK

11110011100001101111 → Digital Modulator → ?

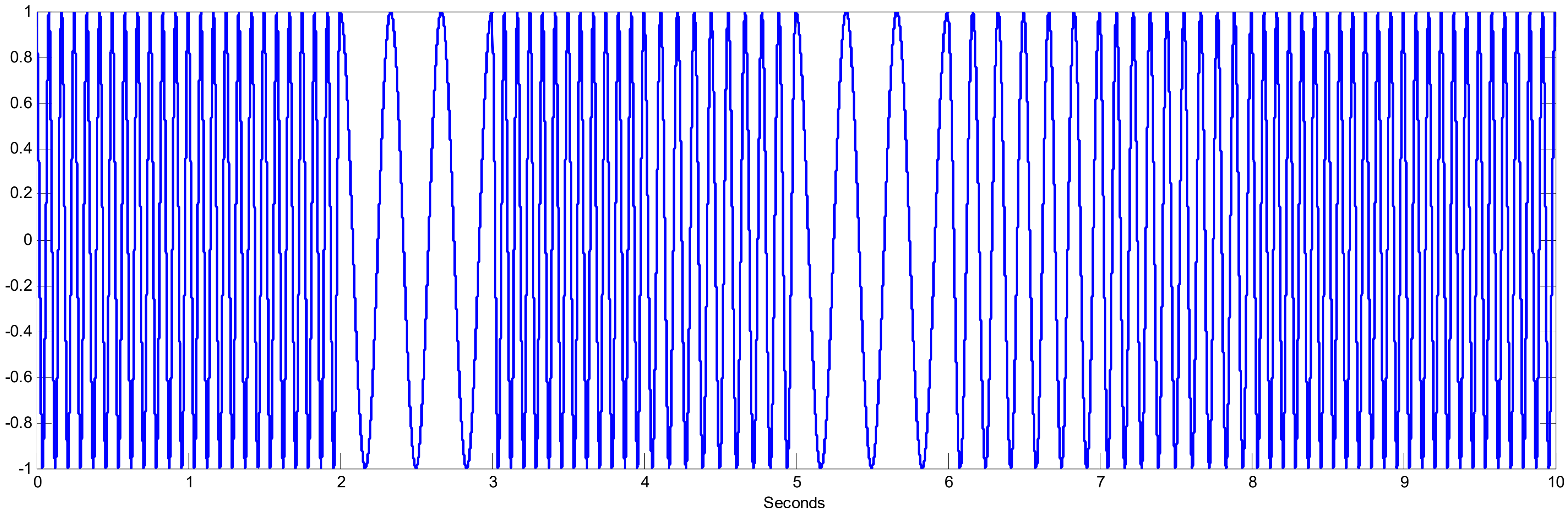
M = 4

$f_c \in \{f_1, f_2, f_3, f_4\} = \{3, 6, 9, 12\}$ [Hz]

[11 11 00 11 10 00 01 10 11 11]

↓

$f_c =$ [12 12 3 12 9 3 6 9 12 12] Hz




[FSK_playTones_Demo.m]

# FSK

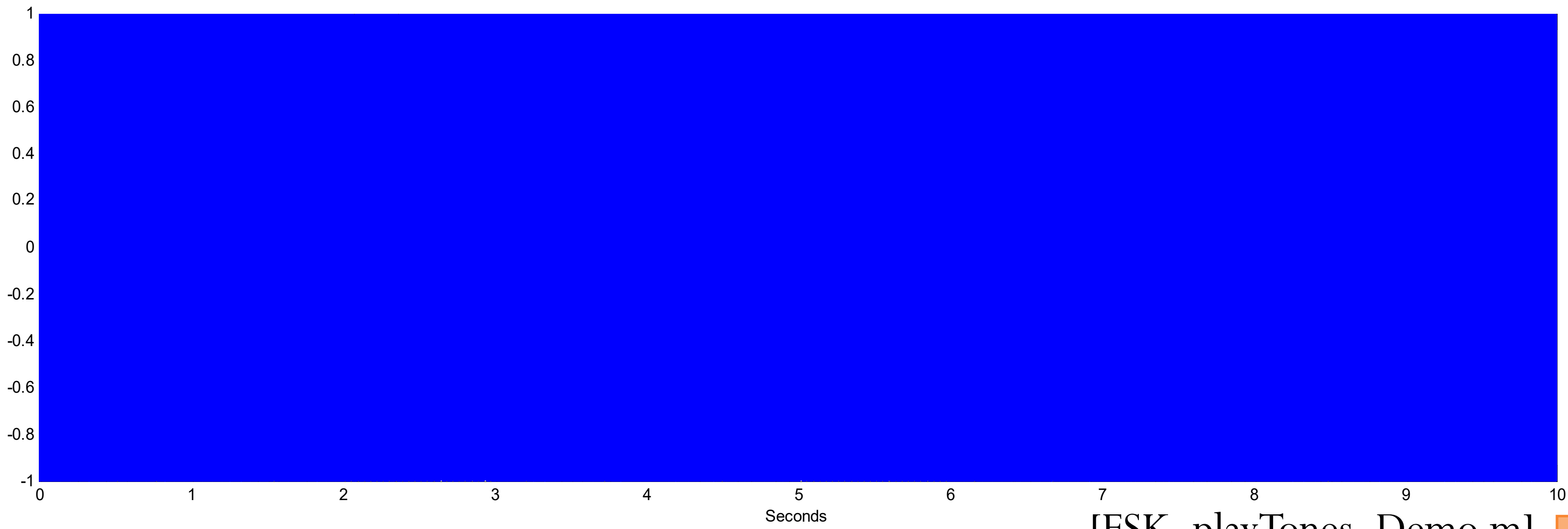
11110011100001101111 → Digital Modulator → ?

M = 4

$f_c \in \{f_1, f_2, f_3, f_4\} = \{100, 200, 300, 400\}$ [Hz]

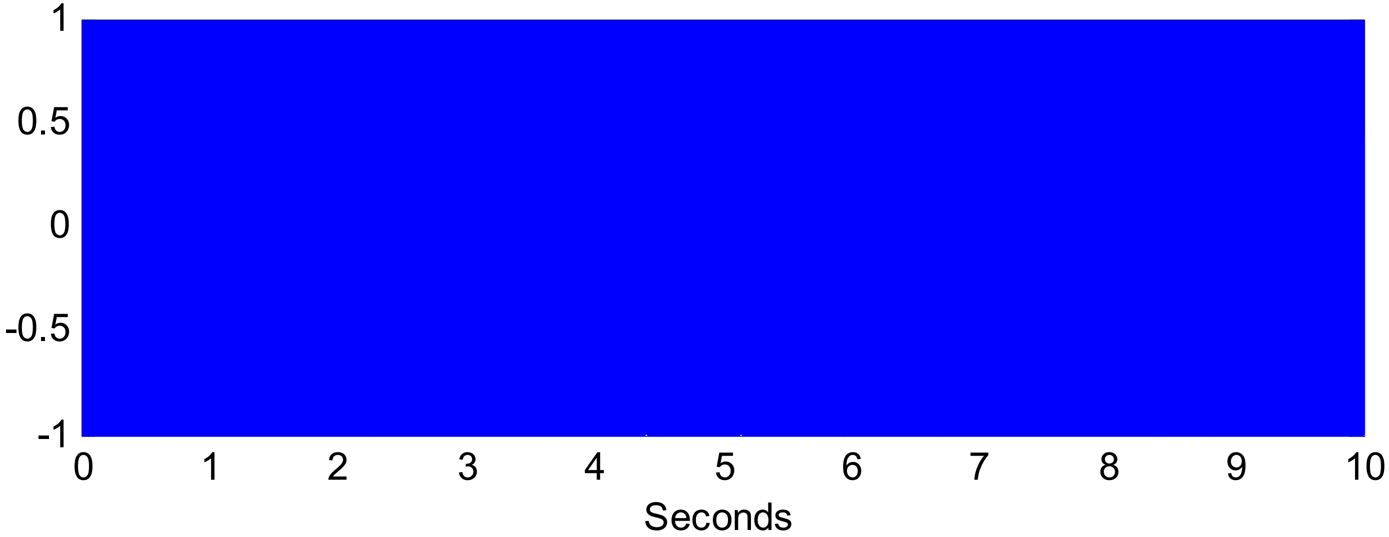
[11 11 00 11 10 00 01 10 11 11]

$f_c$ = [400 400 100 400 300 100 200 300 400 400] Hz




[FSK_playTones_Demo.m]

# FSK


$R_s = 5$

Bit rate = 10 bps

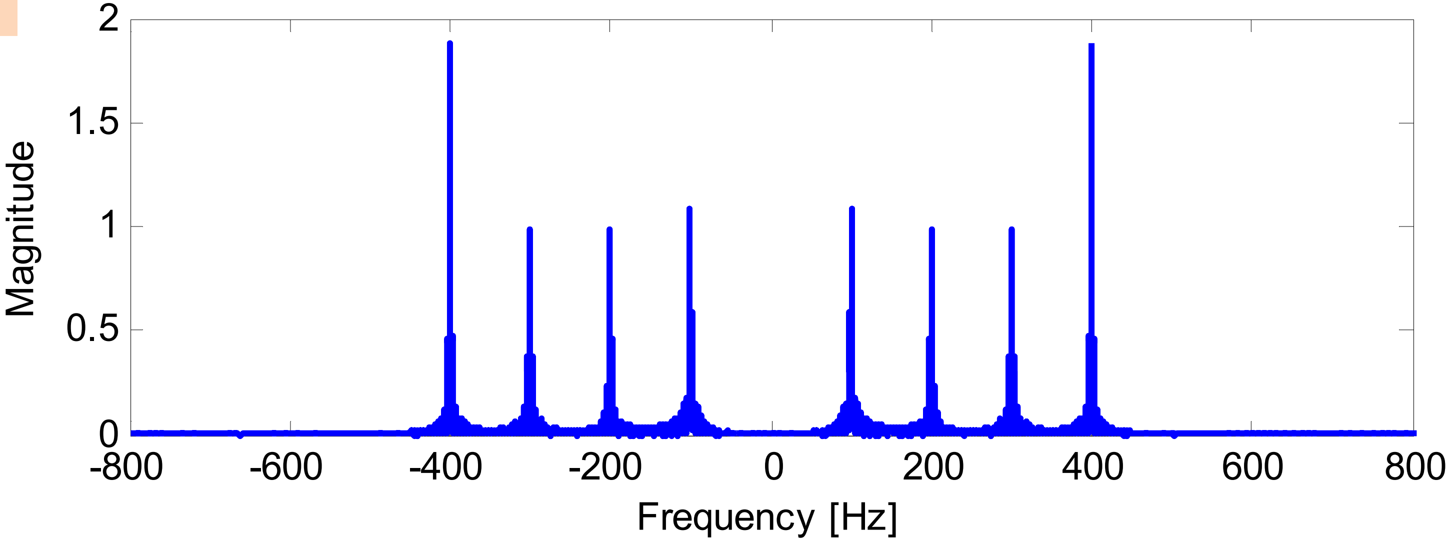
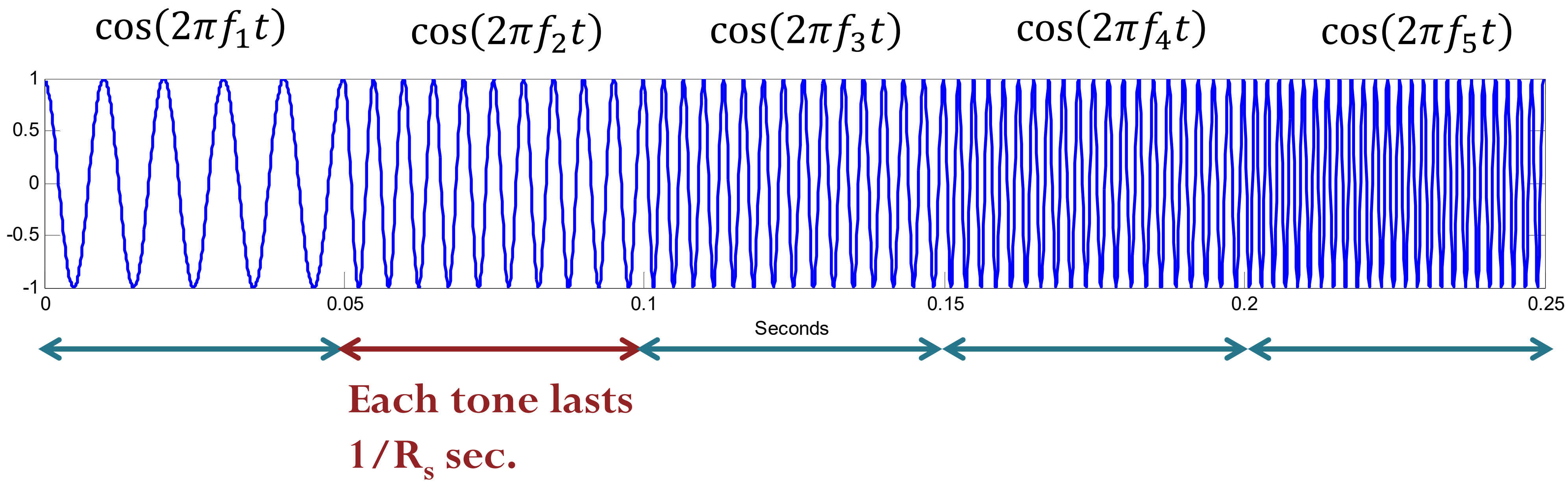


Figure 41

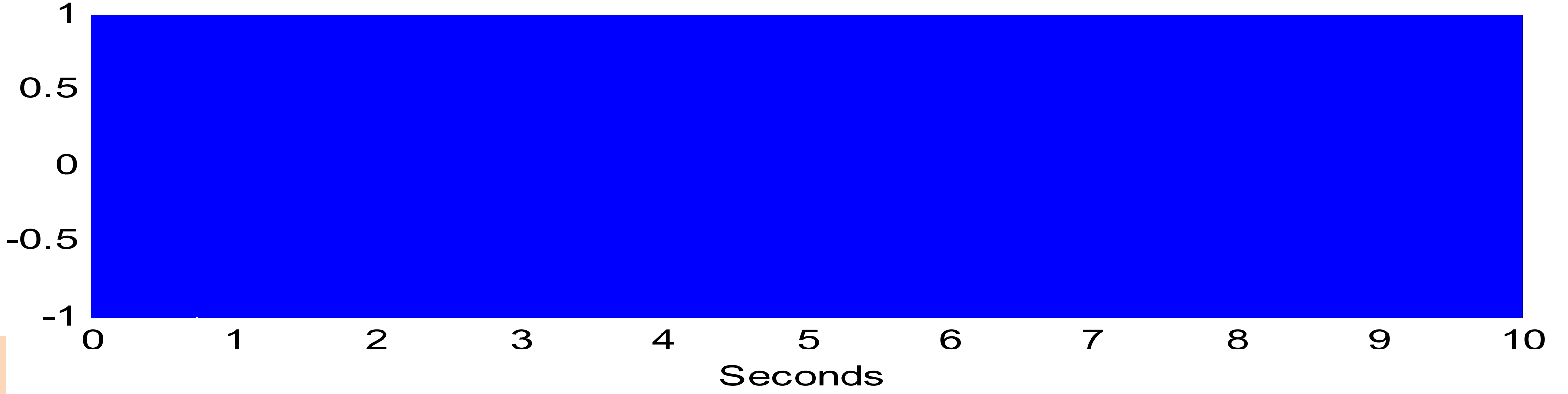
# Five Frequencies



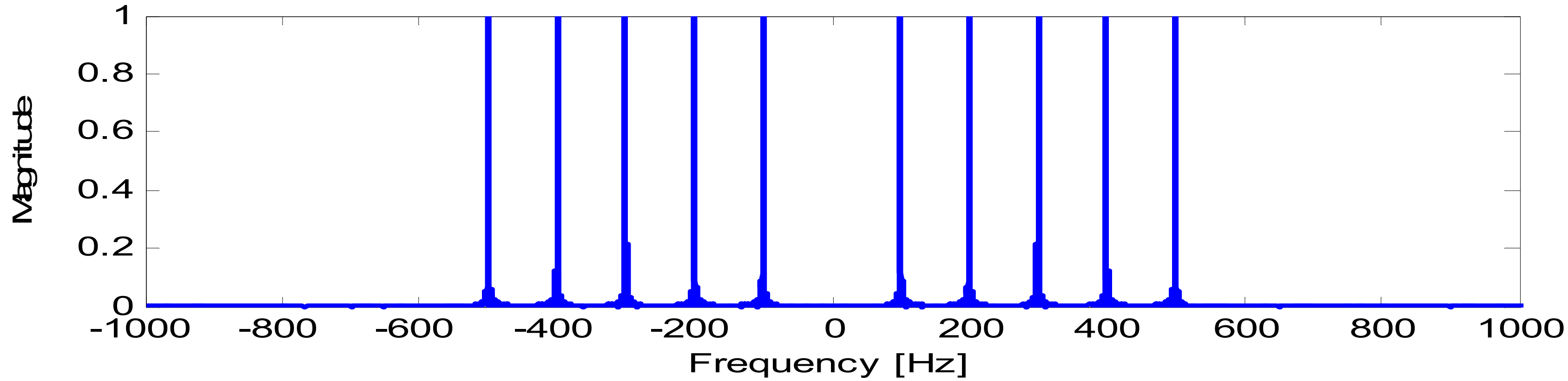

Rate = $R_s$ frequency-changes per second

# Spectrum of Five Frequencies (1/5)

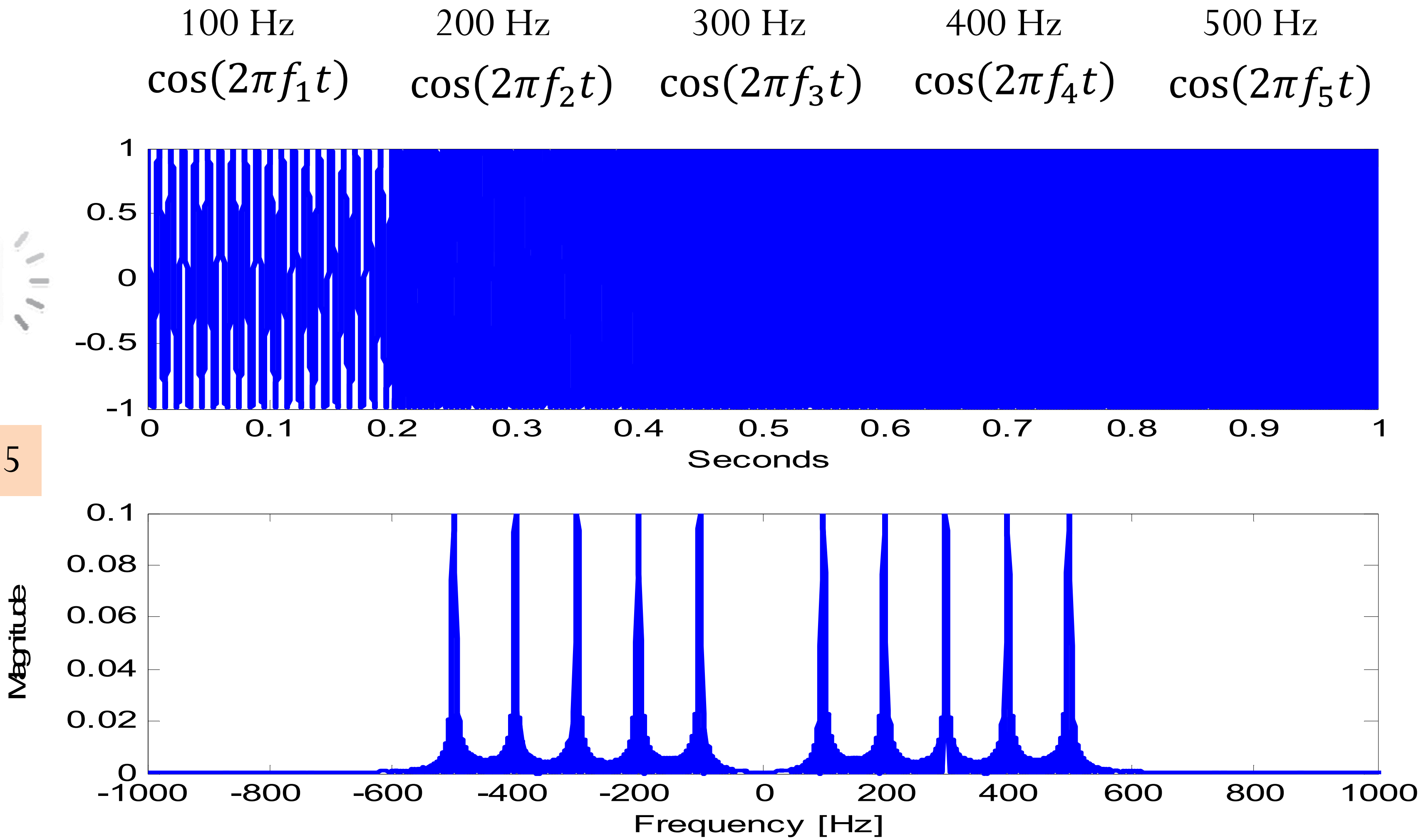
| 100 Hz | 200 Hz | 300 Hz | 400 Hz | 500 Hz |
| --- | --- | --- | --- | --- |
| $\cos(2\pi f_1 t)$ | $\cos(2\pi f_2 t)$ | $\cos(2\pi f_3 t)$ | $\cos(2\pi f_4 t)$ | $\cos(2\pi f_5 t)$ |




$R_s = 0.5$



# Spectrum of Five Frequencies (2/5)




$R_s = 5$

# Cos vs. Cos Pulse

$$x(t) = \cos(2\pi(100)t)$$

$$x(t) = \begin{cases} \cos(2\pi(100)t), & 0.4 \le t \le 0.6, \\ 0, & \text{otherwise}. \end{cases}$$

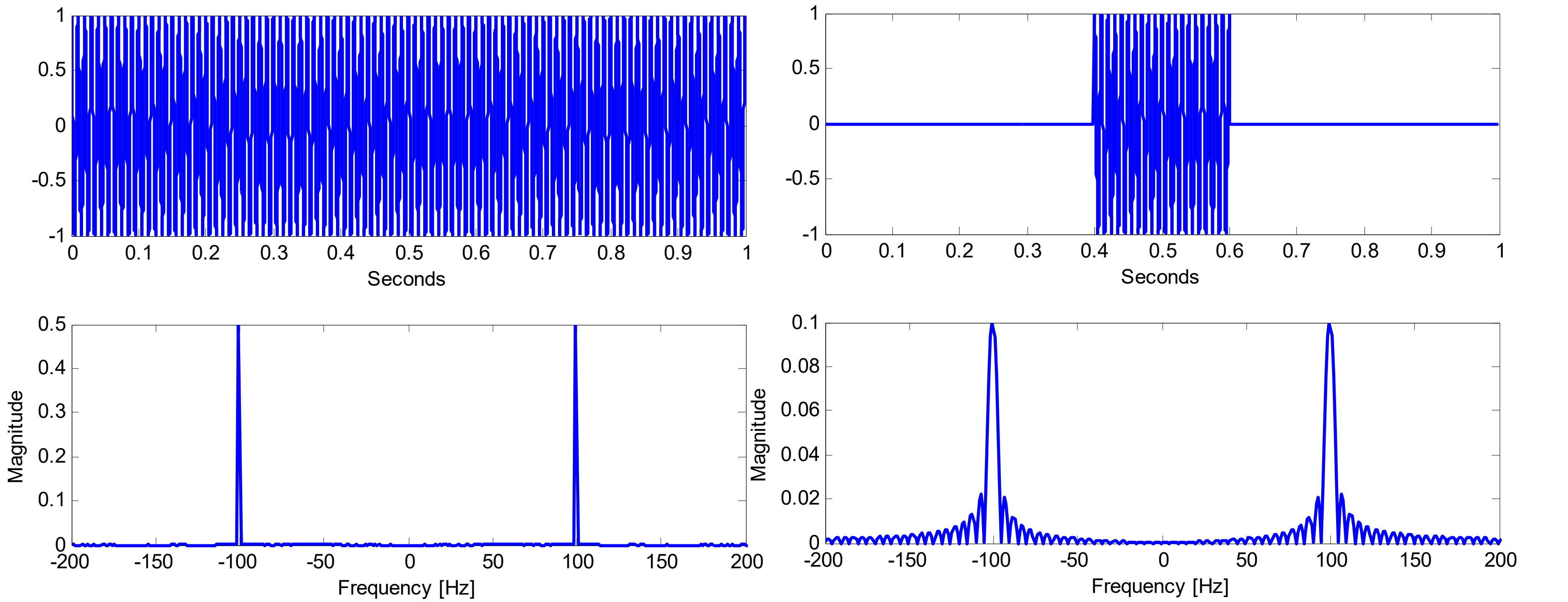


Figure 42

# Cosine Pulse

$$x(t) = \begin{cases} \cos(2\pi(100)t), & 0.5 \le t \le 0.6, \\ 0, & \text{otherwise.} \end{cases}$$

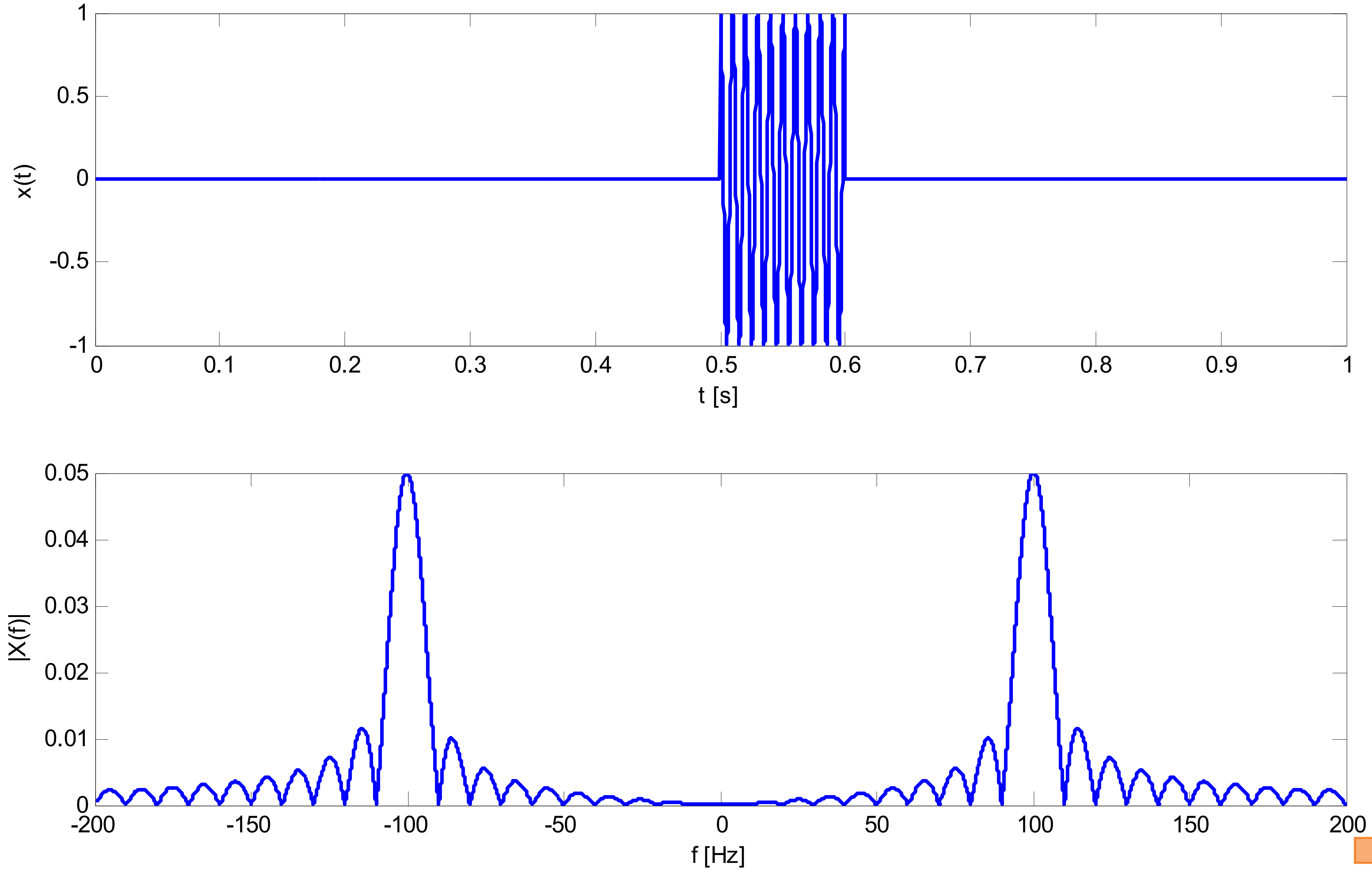
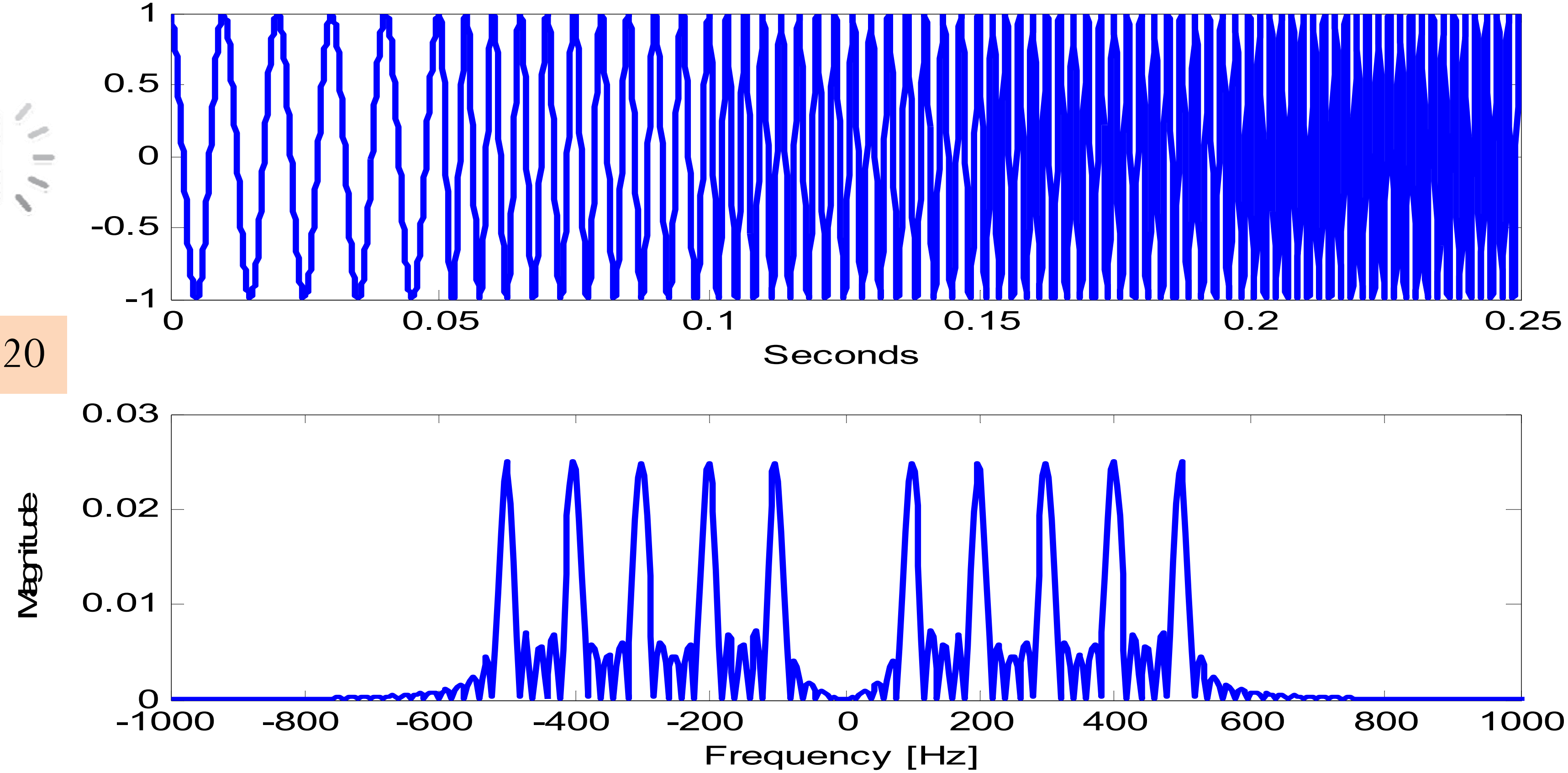


Figure 43

# Spectrum of Five Frequencies (3/5)

| 100 Hz | 200 Hz | 300 Hz | 400 Hz | 500 Hz |
|---|---|---|---|---|
| $\cos(2\pi f_1 t)$ | $\cos(2\pi f_2 t)$ | $\cos(2\pi f_3 t)$ | $\cos(2\pi f_4 t)$ | $\cos(2\pi f_5 t)$ |

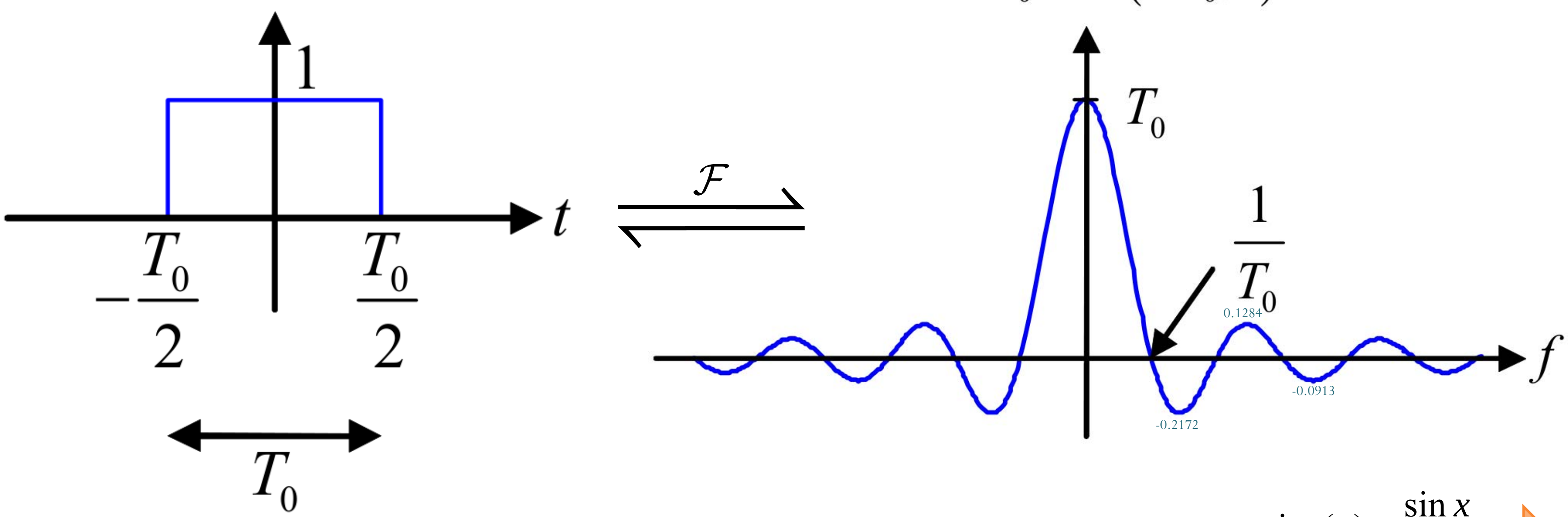
$R_s = 20$



# Fourier Transform Pairs (2)

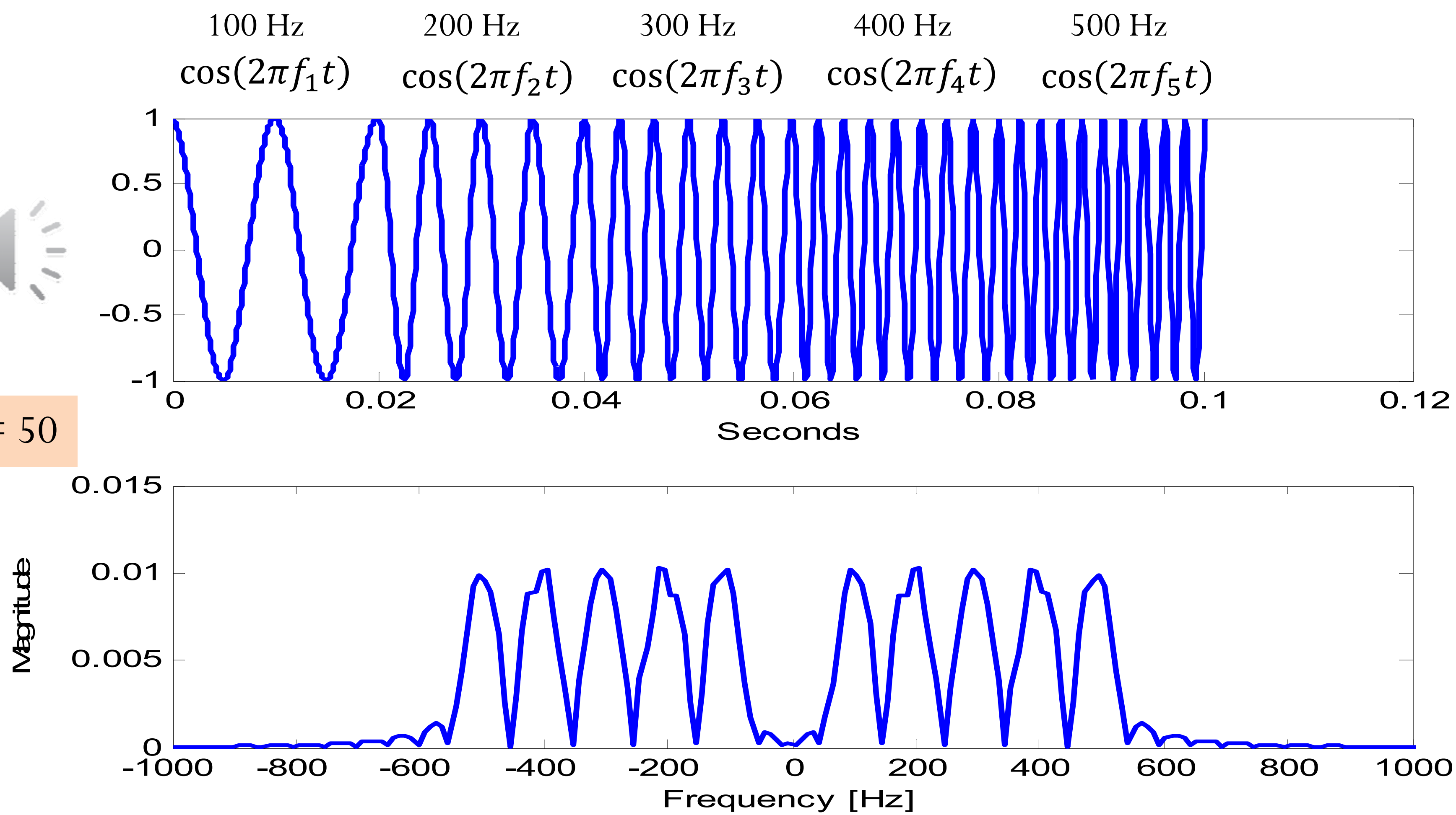
Time Domain      Frequency Domain

$$g(t)=\int_{-\infty}^{\infty} G(f)e^{j2\pi ft}df \overset{\mathcal{F}}{\rightleftharpoons} G(f)=\int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt$$




$$\text{sinc}(x)=\frac{\sin x}{x}$$

# Spectrum of Five Frequencies (4/5)




$R_s = 50$

# Spectrum of Five Frequencies (5/5)

| 100 Hz | 200 Hz | 300 Hz | 400 Hz | 500 Hz |
|---|---|---|---|---|
| $\cos(2\pi f_1 t)$ | $\cos(2\pi f_2 t)$ | $\cos(2\pi f_3 t)$ | $\cos(2\pi f_4 t)$ | $\cos(2\pi f_5 t)$ |

$R_s = 0.5$